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DEALING WITH TECHNICAL PROBLEMS
RELATING TO THE PRODUCTS, PROCESSES AND INVESTIGATIONS OF
THE PHILIPS INDUSTRIES

Photo Hans de Boer

THE PHILIPS PAVILION AT THE 1958 BRUSSELS WORLD FAIR

I. THE ARCHITECTURAL DESIGN OF LE CORBUSIER AND XENAKIS
II. THE HYPERBOLIC-PARABOLOIDAL SHELL AND ITS MECHANICAL PROPERTIES
III. MODEL TESTS FOR PROVING THE CONSTRUCTION OF THE PAVILION
IV. CONSTRUCTION OF THE PAVILION IN PRESTRESSED CONCRETE

061.41(493.2):725.91

At the Brussels World Fair, near the Dutch section, Philips have had their own pavilion built. Visitors to the pavilion are entertained to a "spectacle in light and sound", the object of which is to demonstrate the capabilities of modern technology in some of Philips' major fields of endeavour — illuminating engineering, electro-acoustics, electronics and automatic control techniques — and also to give an impression of the way in which these technical facilities may in the future be turned to artistic ends. The basic conception was propounded by Mr. L. C. Kalff, Arts Director of Philips, and the architect Le Corbusier was commissioned to give effect to it. The latter wished not only to design the building but also wrote the scenario for the spectacle, which he has entitled "An Electronic Poem". The music for the spectacle was composed by Edgar Varèse. Completely automatic performances of the spectacle are now being given scores of times a day, controlled by a magnetic tape with fifteen command tracks.

A later article in this Review will be devoted to the performance, *which is produced by film projectors, lamps and hundreds of*

loudspeakers, and to the technical devices and methods used. The articles in the present issue are devoted to the pavilion itself. From the outset this pavilion, designed by Le Corbusier and his collaborator Y. Xenakis, has aroused considerable interest in the world of architecture because of its extraordinary conception and advanced design as a shell structure. The building is entirely composed of shells having the form of hyperbolic paraboloids. The method of construction in prestressed concrete, proposed and translated into reality by Dr. H. C. Duyster, director of the contracting firm N.V. "Strabed" and a specialist in this field, is remarkable for its originality and elegance. Before plunging into this adventure — as Mr. Duyster himself put it — N.V. "Strabed" approached Professor C. G. J. Vreedenburgh of the Delft Technische Hogeschool for advice concerning the stresses that might occur in the shells when loaded by their own weight, and by wind and snow loads. To satisfy N.V. Strabed as to the feasibility of the proposed scheme of construction and supply data for the actual structure, tests on scale models were made by Mr. A. L. Bouma and Mr. F. K. Ligtenberg in the "T.N.O." Institute at Rijswijk (Netherlands) and the Stevin Laboratory at Delft.

These aspects are treated in the four articles printed in this issue: the architect's conception, the mechanical principles, the model tests and the actual construction of the building.

*As regards the first article it should be mentioned that Y. Xenakis, the architect largely responsible for designing the shape of the pavilion, has placed at our disposal a description of the way in which the architectural design of the building was evolved. In our opinion, however, there was little point in attempting to render the author's French text faithfully into English or other languages, for translation would do less than justice to the eloquence of the artist's highly individual style and risk distorting the sentiments of the original. It was therefore decided to confine the English rendering of his article to a reproduction of the factual contents *).*

The second article in this series also calls for some comment. Although Professor Vreedenburgh has kindly taken great pains to make the train of thought in his text as comprehensible as possible to the readers of this Review, we cannot disguise the fact that many readers will perhaps have difficulty in following the details of his article, lying as it does far outside the range of subjects normally dealt with in these pages. On the other hand, the article should be of particular interest to the specialist, since it provides for the first time in published form certain formulae and results concerning hyperbolic-paraboloidal shells which can be turned to practical architectural use.

*) For those readers who would like to have a copy of the original French text, reprints will be available of the article published in the French edition of this Review.

I. THE ARCHITECTURAL DESIGN OF LE CORBUSIER AND XENAKIS

after Y. XENAKIS †).

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A report is given below of the ideas embodied in the architectural conception of the Philips pavilion and of the various stages through which the design passed before the pavilion acquired its final shape. This report is an authorized shortened version of an article by Y. Xenakis, who also provided the drawings illustrating the evolution of the design. These drawings are the main feature of the article.

When Le Corbusier, in the beginning of 1956, agreed to undertake the design of the Philips pavilion, he had in mind a structure to enclose a space of unconventional form and to be materialized by casting cement on a metal-gauze framework suspended from scaffolding. The structure would have a roof and surfaces on which pictures, colours and film scenes could be projected for performing a spectacle in light and sound — a so-titled "Electronic Poem". In October 1956, Y. Xenakis, under the direction of Le Corbusier, entered upon a detailed study of the project [1]). The result was a design based entirely on the use of *ruled surfaces*.

This result, to which artistic intuition as well as practical considerations contributed, will be elucidated in the following pages.

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The first design

The *ground-plan* of the pavilion was fairly simply established, being dictated by the requirements for the performance of the "Electronic Poem". Each performance was to last 8 to 10 minutes and to be attended by some 600 or 700 persons, uniformly distributed over the whole floor surface of the pavilion. A space of more or less circular plan was therefore needed, with an area of 400 or 500 m^2 and with two large "spouts" as entrance and exit channels.

1) A brief account of this study has already been published: Y. Xenakis, Le Corbusier's "Elektronisches Gedicht" und der Philips Pavillon, Gravesaner Blätter 3, 47-54, 1957 (No. 9).

In order to be able to produce various fantastic effects, locally changing colours, shifts of light and shade, etc. in the projection of pictures or colour slides, the enclosing walls (or at least part of them) had to be *curved* surfaces, so that they would receive the light from divergent angles. All uniformity was to be avoided, even the uniformity of curvature found in spherical and cylindrical vaults. This led to the idea of having surfaces with differing radii paraboloid (hypar) is also produced by moving a straight line such that it always remains parallel to a given plane, but in this case it slides along two skew straight lines (rectilinear directrixes). The static stress distribution in a shell having the form of a hyperbolic paraboloid can, to a certain extent, be calculated: such a shell is found to possess remarkable properties of strength and stability (see article II in this series). Moreover, these surfaces

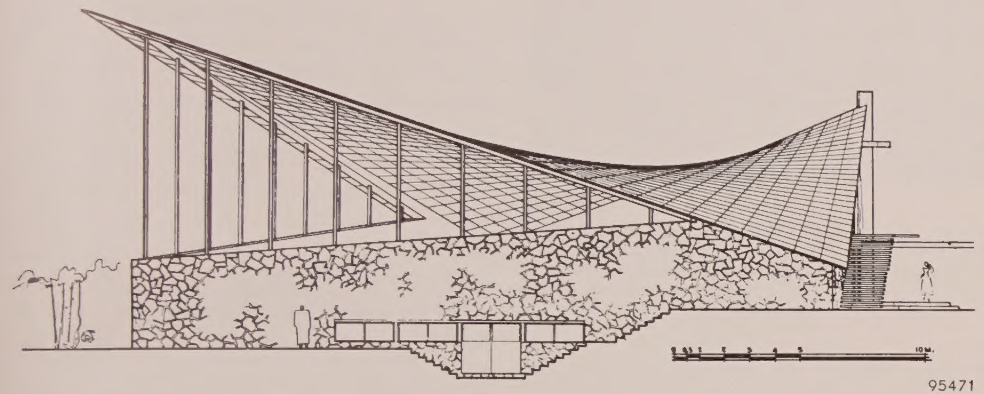

Fig. 1. The church Notre Dame de la Solitude at Coyoacan, Mexico, having a concrete shell roof in the form of a hyperbolic paraboloid, designed by the architect Felix Candela. (Illustration from: F. Candela, Les voûtes minces et l'espace architectural, L'architecture d'aujourd'hui **27**, 22-27, March 1956.)

of curvature. Such surfaces also seemed suitable for meeting the *acoustic* requirements. To allow complete freedom for creating a wide variety of spatial impressions with the aid of loudspeakers, the aim was to avoid as far as possible the uncontrolled acoustic contributions due to reflections from the walls and which are audible either as isolated echos or as reverberation. It is known that parallel flat walls are dangerous in this respect, because of repeated reflections; parts of spherical surfaces are equally inappropriate, since they can give rise to localized echos.

Having turned his thoughts to surfaces with widely varying radii of curvature, Xenakis was led naturally to consider saddle surfaces, and in particular the *ruled surfaces* that come into this category. Through the work of Laffaille and other pioneers in this field, the architect was familiar with simple ruled surfaces, such as the hyperbolic paraboloid and the conoid. The conoid is obtained by letting a straight line (a generator) slide along two non-intersecting lines (directrixes), one a straight line and the other an arbitrary curve, such that it remains parallel to a given plane. The hyperbolic produced by straight lines readily lend themselves to construction in straight wooden beams or in concrete (see article IV). These attractive properties have led to an increasing use of such shell structures in various countries, particularly for roof constructions (*fig. 1*).

The Philips pavilion offered the architect a unique opportunity to build a structure entirely from these ruled surfaces, and in this way to create a homogeneous three-dimensional envelope in the sense that the three dimensions would each really play an independent role, as opposed to conventional architecture in which, usually, the form of the ground-plan is still manifest in every section of the building high above the ground.

The working-out of this novel architectural idea, however, was necessarily a process involving artistic intuition and a feeling for form rather than a question of reasoning. The series of sketches, *figs. 2-10*, allow the architect to show how he arrived at his first design.

This design (fig. 10) contains a conoid E, a surface consisting mainly of two conoids A and D, two hyperbolic paraboloids K and G, a connecting cone

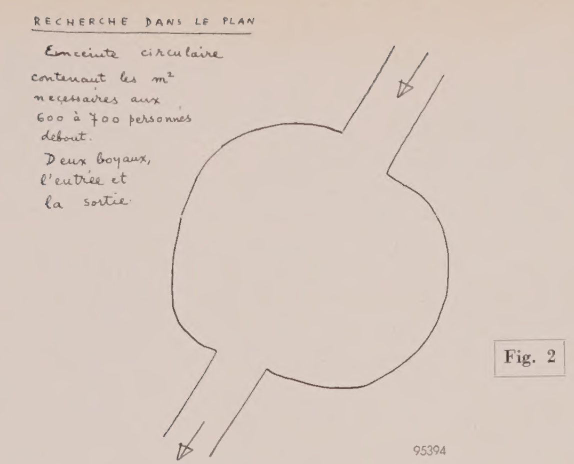

Fig. 2

Fig. 3

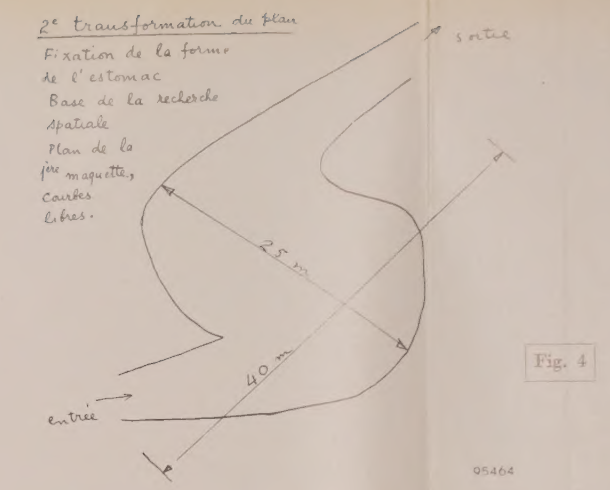

Fig. 4

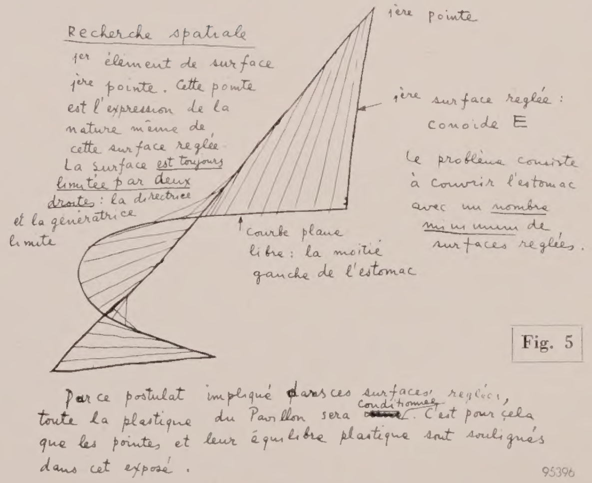

Fig. 5

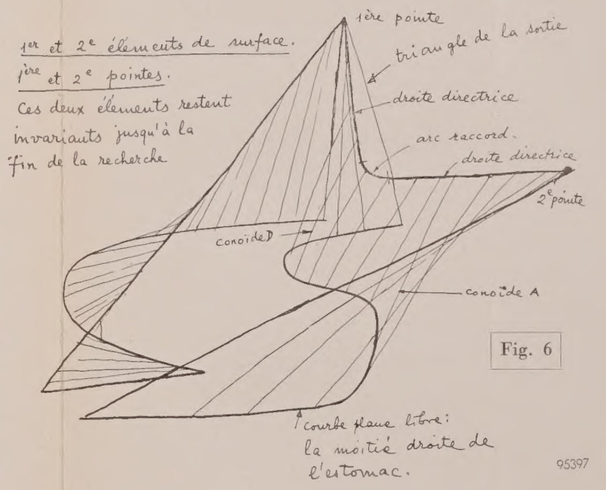

Fig. 6

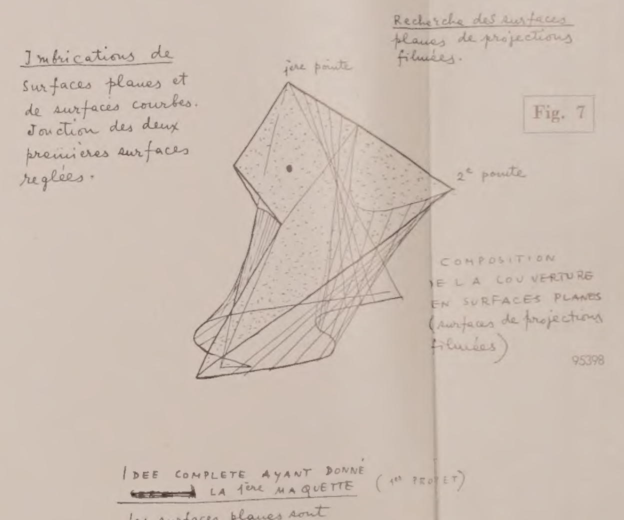

Fig. 7

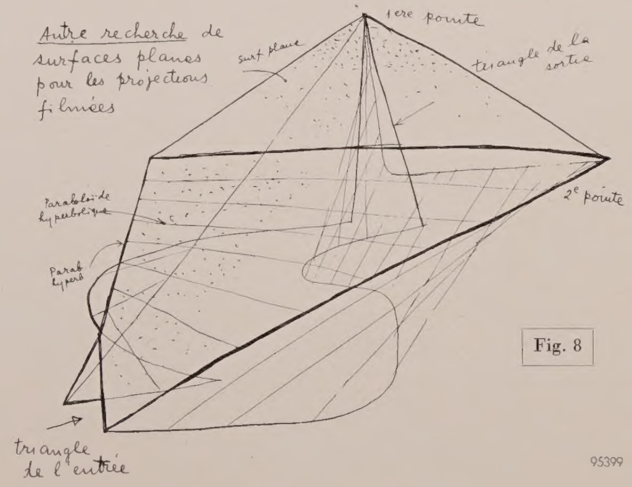

Fig. 8

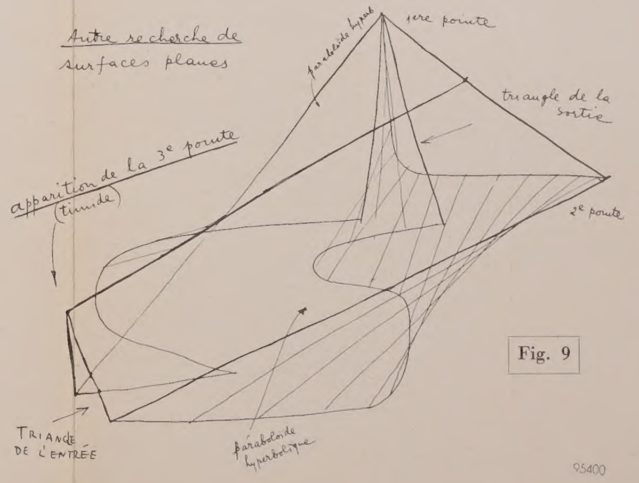

Fig. 9

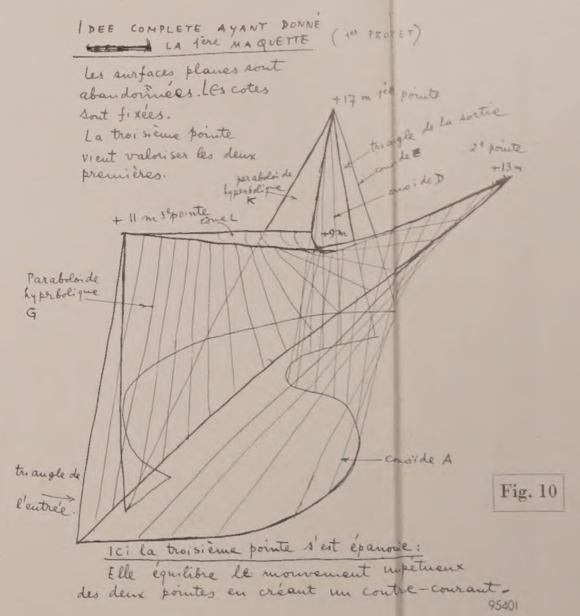

Fig. 10

Figs. 2-4. Development of the ground-plan.

Fig. 2. Circular space with two "spouts" as entrance and exit channels.

Fig. 3. Further development of the plan form; partly from its shape and partly because of its function, the architect refers to it succinctly as "l'estomac".

Fig. 4. The ground-plan forming the basis of the first design.

Figs. 5-10. Stages in the development of the first design.

Fig. 5. Ground profile of the *left half* of the "stomach". The intention was to build over the ground-plan a shell composed of as few ruled surfaces as possible. A conoid (E) is constructed through the ground profile curve; this wall is bounded by two straight lines, viz. the straight directrix (rising from the left extremity of the ground profile) and the outermost ruling line (passing through the right extremity of the ground profile). This produces the first "peak" of the pavilion.

Fig. 6. A ruled surface, but consisting of *two* conoids, A and D, is also laid through the curve bounding the *right half* of the "stomach". The straight directrix of D passes through the first peak, and the outermost ruling line at this side forms with that of E a triangular exit. The straight directrix of A passes through a second peak and is joined by an arc to that of D.

This basic form is that used in the first design and was retained, with some modifications, in the final structure. The main problem of the design was to establish an aesthetic balance between the two peaks.

Fig. 7. Attempt to close the space between the two ruled surfaces of the first design by flat surfaces (which might serve as projection walls).

Fig. 8. Another attempt. Above the entrance channel a small triangular opening is formed, flanked by two hyperbolic paraboloids (later denoted by G and K), and the whole is covered with a horizontal top surface.

Fig. 9. Elaboration of fig. 8. The third peak begins tentatively to take shape.

Fig. 10. The first design completed (see also the first model, fig. 11). There are no longer any flat surfaces. The third peak is fully developed and creates, with its opposing sweep, a counterbalance for the first two peaks. The heights of the three peaks have been established. The third peak and the small arc connecting the straight directrixes of conoids A and D (see fig. 6) form, respectively, the apex and the base of a part of a cone L.

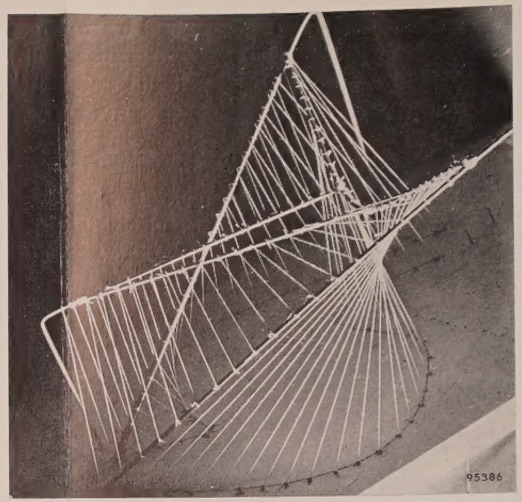

Photo Lucien Hervé

Fig. 11. The first model. The "stomach" is set out on the base of the model; the strings indicate the ruled surfaces. The intersections of the ruled surfaces are represented by spokes of piano wire. Their bent-over ends have no structural significance.

L and two open triangles as entrance and exit. The two peaks, produced from the oblique straight lines arising out of one of the channels (fig. 6) are counterbalanced by a third peak projecting above this channel.

Fig. 11 shows a model of the first design. The ribs in which the surfaces intersect are formed in this model by spokes of piano wire, the bent ends being anchored in a wooden base. The surfaces are produced by spanning strings between the ribs.

The second design

At this stage, engineers of a Parisian firm of contractors were consulted by the architects regarding the system of construction.

With a view to soundproofing, Philips had specified a wall weight of 120 kg/m^2 (concrete or cement about 5 cm thick). There was therefore no question of building the pavilion in the form of a tent, whether or not with metal-reinforced "canvas". The engineers consulted believed that in these circumstances the pavilion would have to be constructed on a fairly heavy metal skeleton, after the manner of the wire spokes in the model and with supporting stanchions corresponding to the vertical bent wires in the model. At all events they thought it desirable to change from conoids to hyperbolic paraboloids so as to make it possible to specify more easily the exact curvature of all surfaces and simplify the calculation of static stresses as well as the work of erection.

This advice was accepted by Le Corbusier and Xenakis, especially since they themselves felt that the first design had certain aesthetic weaknesses which in any case called for modification.

Xenakis set about converting the surfaces by experiment. His method was simple: he used two straight metal spokes joined by a system of elastic strings fixed at equidistant points along each spoke. The strings formed the ruling lines of a hypar, whose geometry was determined by the distance between the spokes, the angle between them and the positioning of two arbitrary strings. Other variables determine the position of the hypar with respect to ground level. To select each of the pavilion surfaces the architect had to proceed by trial and error, simultaneously varying all the above variables; as soon as he found a satisfactory form for a particular surface, he immediately put it down on paper in the form of an orthogonal projection [2]). For this purpose it is sufficient to give horizontal and vertical projections showing the positions of the two spokes and of two pairs of corresponding points thereon (e.g. the end points of the two outermost strings on the spokes; see *figs. 12* and *13*). This done, *all* pairs of corresponding points are fixed, each pair defining a ruling line. The points at which the ruling lines meet the horizontal plane give the intersection of this plane with the part of the hypar surface involved (*fig. 14*). This intersection can be part of a hyperbola or of a parabola (this is the case when one of the spokes is *below* the horizontal plane), or it can be a straight line (one of the spokes lies *in* the horizontal plane), or, in special cases, it can be a single point. There are also some hypar shells in the design, of which the part of the surface used does not touch the ground at all.

The first step in revising the original design was to change the position in space of the three peaks so as to obtain more harmonious proportions. The difference between the second and third peak had to be accentuated, and the middle cone *L* widened. The architect now fixed the height of the peaks at 21 m, 13 m, and 18 m respectively. He then proceeded, by alternately experimenting with the spoke and string model and drawing the surfaces found, to establish the hyperbolic paraboloidal surfaces that both gave an aesthetically satisfying form and yielded intersections with the ground level which were as much as possible in keeping with the original ground-plan.

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[2]) It is clear that it would hardly be convenient to define the hypar surfaces in terms of the numerical values of the coefficients in the appropriate equation of the surface (see II).

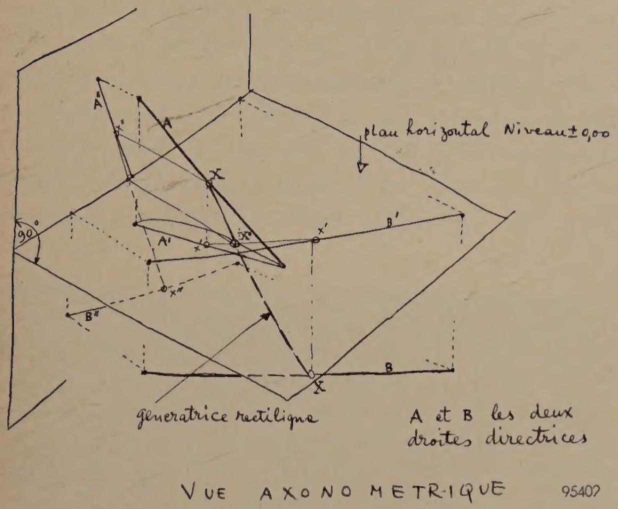

Fig. 12

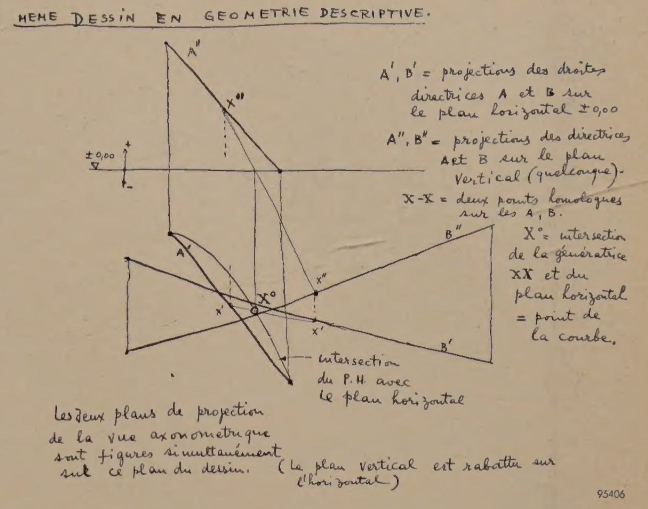

Fig. 13

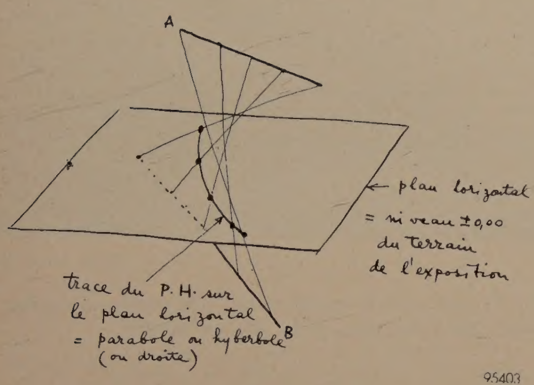

Fig. 14

Fig. 12. Isometric drawing to indicate how the orthogonal projection (fig. 13) of a hyperbolic paraboloid may be constructed. The two directrixes A, B are projected on to a horizontal plane (A', B') and on to a vertical plane (A'', B''); two pairs of points, viz. the end points of A and B, and their corresponding projections are shown.

Fig. 13. Horizontal and vertical projections of the directrixes of a hyperbolic paraboloid from fig. 12. Also shown are the projections $X'X'$ and $X''X''$ of an arbitrary generator line connecting two corresponding points X, X on the directrixes. This generator passes through the horizontal plane at point X^0.

Fig. 14. The intersection of a hyperbolic paraboloid with the horizontal plane is constructed from the points at which a series of generator lines intersect this plane.

By December 1956 the second design had been completely worked out in this way and set down on paper (*figs. 15* and *16*). From this design a new model was made (*fig. 17*).

Comparing the second with the first design, we see that the hypars G and K (which form the most important surfaces for the projection of pictures) have been retained, but the cone L has been widened and the conoids A, E and D changed into five hypars A, E and B, N, D. In addition, two new hypars C and F appear. Surface F, which abuts on E, provides the necessary space for certain installations (air-conditioning plant, toilets, control room) and for the extensive equipment needed for automatically performing, several times an hour, the spectacle of light and sound.

Final modifications

Most of the contracting firms approached by Philips at this stage of the design had only more or less conventional schemes of construction to propose, which were conspicuously out of keeping with the revolutionary style of the structure. Double-walled shells were suggested, having a total thickness of

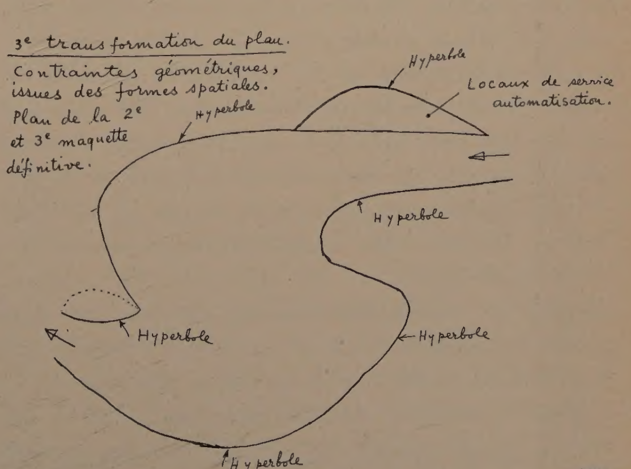

Fig. 15. Revised ground-plan for the second (and definitive) design. The bounding curves are now composed of parts of hyperbolae (for practical reasons the entrance and exit were interchanged with respect to the first design).

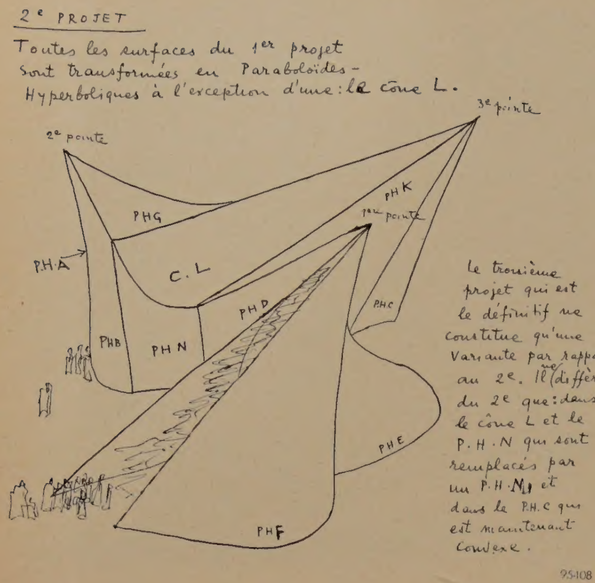

Fig. 16. The second design. All surfaces of the first design, except cone L, have now been converted into hyperbolic paraboloids, and two new hyperbolic paraboloids (F and C) have been introduced. Compared with fig. 10, the design is seen here from the opposite side, as can be seen from cone L, the apex of which appears top right in this sketch. The first peak is here in the foreground.

80 cm and made of wood, metal or plaster carried by fairly complex skeleton structures. The only proposal that was really in unison with the architect's intentions, while being at the same time reasonable in price, came from the Belgian contracting firm N.V. "Strabed", directed by Dr. H. C. Duyster. Mr. Duyster's plan was to build the pavilion as a shell structure of prestressed concrete 5 cm thick, which would be largely self-supporting, i.e. only a few stanchions would be used merely to give the walls some additional support. The intention was to follow closely the form of the second design, with only one minor modification. The latter arose from a misunderstanding of the architect's drawing, in which the hyperbolic paraboloids that did not touch the ground were indicated only summarily, leading Mr. Duyster to interpret the cone L and the hypar N (fig. 16) as parts of a single hypar (denoted M below). In fact, this simplification improved the geometrical purity of the structure. The elegant method by which Mr. Duyster proposed to construct the ruled surfaces of the pavilion in concrete is described in the fourth article of this series.

Finally, another modification was decided on, which, though a minor one in its effect on the strength of the structure, was of the utmost importance as regards the overall architectural effect. The design still envisaged supporting stanchions, one of which was actually inside the enclosed space, and as such was a nuisance. The architect Xenakis now proposed a slight change in the new hypar M and in B in order to make it possible to dispense with the stanchions entirely. The reasoning was that the edge members (ribs) at the relevant shell intersections ought to be able to take over, at least for the greater part, the supporting function of the stanchions.

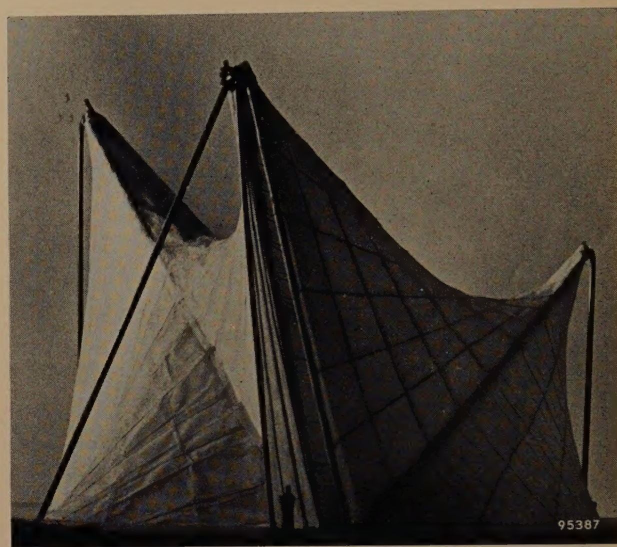

Photo Lucien Hervé

Fig. 17. Second model, seen from the side which now forms the entrance; the third peak is in the foreground.

The model tests (see article III) confirmed that in the design so modified the stanchions were superfluous. The structure was thus made entirely self-supporting, that is to say it no longer contained supporting elements that were not embodied in

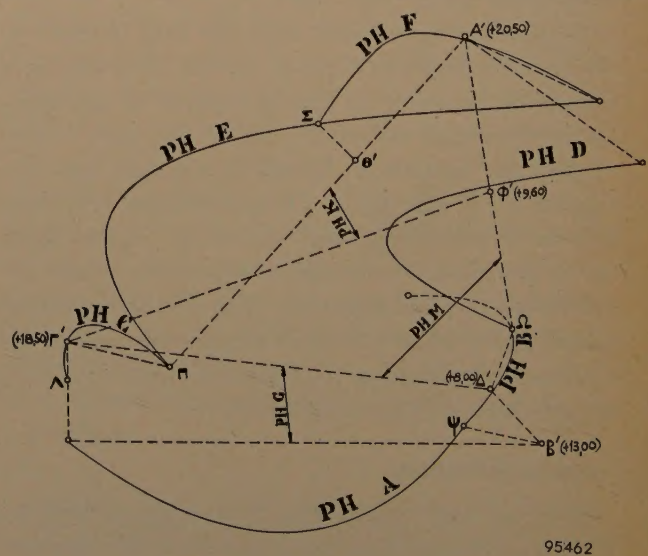

Fig. 18. General plan of the final design (enlarged from fig. 19).

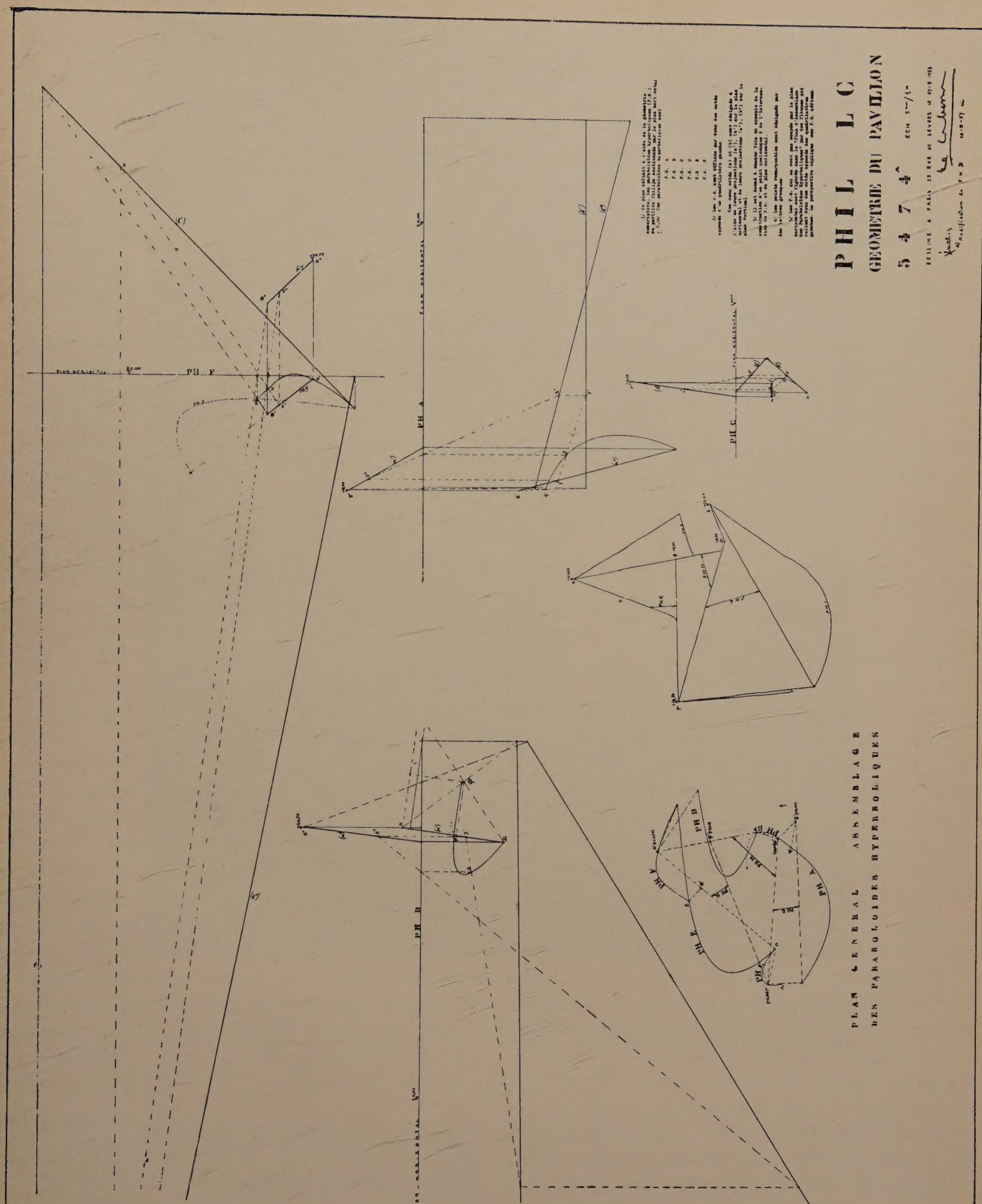

Fig. 19. Part of the scale drawing 1 : 200 of the wall surfaces of the definitive design, signed by the architects Le Corbusier and Xenakis.

the wall surfaces. To strengthen the third peak, which slopes at a very oblique angle, the hypar *C* was made convex at its foot instead of concave, and finally the two triangular openings were partly closed with extra hypars abutting on the existing ones. In this way the definitive form of the pavilion was arrived at, as illustrated in the plans of *figs. 18* and *19*.

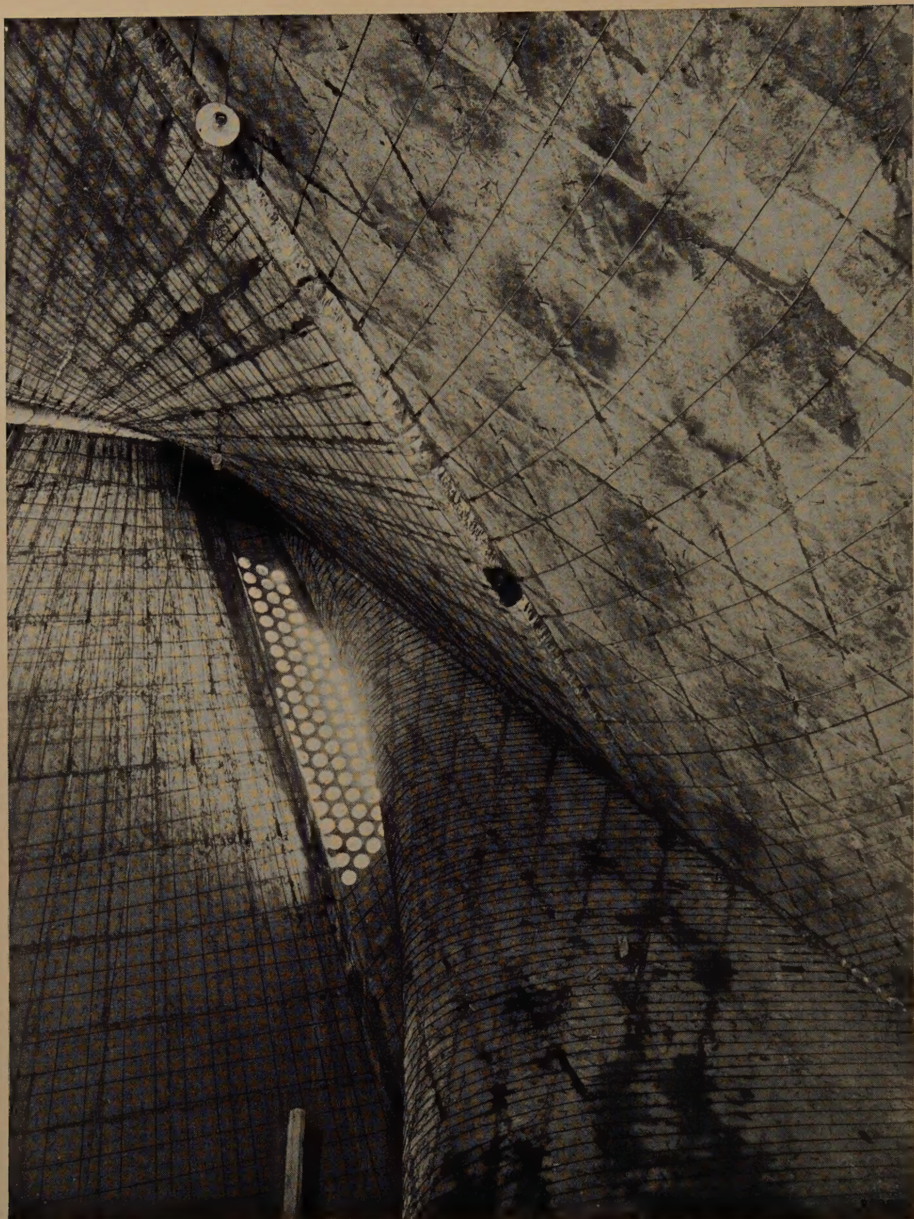

Fig. 20. Photograph of part of the interior of the pavilion. The prestressing wires on the concrete, which enhance the plastic form of the structure, are unfortunately concealed in the finished pavilion by a surfacing required for the projection of colours and pictures.

Fig. 20 shows a photograph of the interior made before the concrete's prestressing wires were concealed by the internal surfacing. This photograph and the title photograph (and also fig. 12 in IV) give an impression of the remarkable plastic figuration of the building.

II. THE HYPERBOLIC-PARABOLOIDAL SHELL AND ITS MECHANICAL PROPERTIES

by C. G. J. VREEDENBURGH *).

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It was about the year 1935 that Laffaille and Aimond published the first studies on the distribution of forces in "hypar" shells, i.e. thin-walled structures having the form of hyperbolic paraboloids [1][2]).

Shells of spherical and cylindrical form have been used for many years but practical interest in the hypar shell dates only from the last ten years. It would seem that saddle surfaces (surfaces having opposite curvatures along different directions) such as these did not find much acceptance because they seemed to defy architectural conventions. Moreover it was thought that hypar shells would be more costly to build than shells of the normal spherical and cylindrical shapes.

Views on this subject have now changed considerably. Partly through the pioneering work of Candela in the U.S.A. [3]) and of Hruban in Czechoslovakia [4]), it became realized that hypar shells not only possess great strength and stability but also lend themselves readily to a synthesis of striking architectural forms in keeping with various tendencies in modern art. The design of Le Corbusier and Xenakis for the Philips pavilion in Brussels which, as described in the first article of this series, is entirely based on hyperbolic paraboloids, has certainly shown that hypar shells can be used for creating the most spectacular architectural fantasies. Furthermore, as regards their actual construction, it is now recognized that hyperbolic paraboloids, because of the two systems of straight lines inherent in them [5]), are particularly well adapted to construction in wood as well as in reinforced or prestressed concrete.

The hypar shell, then, has made its entry into architecture and is being used in many countries and for various kinds of building. However, owing to the relative novelty of this structural form and the greater geometrical intricacy of saddle surfaces, the contractor presented with such an assignment will often be unwilling to rely entirely on experience already available and on his own intuition, but will enlist the aid of a scientific analysis of the expected mechanical behaviour of the structure. It thus came about that, towards the end of January 1957, we were approached by the contracting firm "Strabed" for advice concerning the building of the Philips pavilion.

Now it is simply not possible to calculate exactly the states of stress that can arise in such an exceedingly complicated structure of shells and ribs (the latter at the intersections of the shells). On the basis of theoretical considerations alone we were therefore only able to provide Messrs. Strabed with positive advice of a general nature regarding the feasibility of the architects' design and of the proposed method of building; in order to give a definite answer to certain specific questions recourse was necessary, partly because of the limited time available, to experimental stress analysis, using a model. These tests, which were performed at Rijswijk and Delft (Netherlands) by A.L. Bouma and F. K. Ligtenberg, are described in the third article of this series. Nevertheless it will perhaps be useful to show the interested reader how far it is possible to go with a calculation of the general states of stress in hyperbolic-paraboloidal shells, and to describe the nature of the difficulties which, in intricate cases, oblige one to resort to supplementary tests on a model.

The geometry of the hypar shell

To understand the distribution of forces in a hypar shell it is first necessary to recall some facts about its geometry.

With respect to a rectangular system of axes *Oxyz* (see *fig. 1*), the equation for a hyperbolic paraboloid may be written in the form:

$$z = \frac{x^2}{2r_1} - \frac{y^2}{2r_2}, \quad \ldots\ldots \quad (1)$$

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*) Professor of applied mechanics, Technische Hogeschool, Delft.

[1]) B. Laffaille, Mémoire sur l'étude générale des surfaces gauches minces, Mém. Assoc. Int. Ponts et Charpentes 3, 295-332, 1935.

[2]) F. Aimond, Etude statique des voiles minces en paraboloïde hyperbolique, Mém. Assoc. Int. Ponts et Charpentes 4, 1-112, 1936.

[3]) F. Candela, Structural applications of hyperbolic paraboloidal shells, J. Amer. Concrete Inst., Title No. 51-20, January 1955, pp. 397-415.

[4]) K. Hruban, Obecné řešení žlabových skořepin (The general theory of saddle-surface shells), Institute of Technology Brno, 1953.

[5]) See the third and fourth article of this series. A recent example of a large hypar shell consisting of glued wooden sections is the roof construction of the Information Centre in the Place de Brouckère, Brussels.

where O is the apex of the surface, Oz its axis, and xOz and yOz are planes of symmetry which intersect the hyperbolic paraboloid in the parabolae p_1 and p_2 respectively. The quantities r_1 and r_2 are the radii of curvature of the parabolae p_1 and p_2 at the apex O.

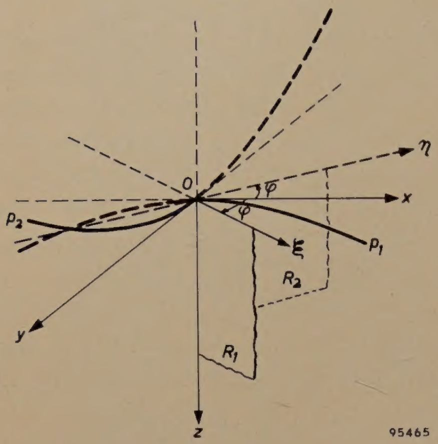


Fig. 1. Geometrical definition of the hyperbolic paraboloid. O = apex; Oz = axis; xOz and yOz are the planes of symmetry; ξOz and ηOz are the directrix planes (R_1 and R_2), to which the two sets of rulings on the surface run parallel.

The plane xOy intersects the surface in the straight lines $O\xi$ and $O\eta$. The axis Ox is the bisector of the angle 2φ between these lines. It can be shown, from (1), that

$$\tan \varphi = \frac{\sqrt{r_2}}{\sqrt{r_1}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

With respect to the system of axes $O\xi\eta z$ (the axes $O\xi$ and $O\eta$ are not, in the general case, perpendicular to each other) the equation for the hypar is:

$$z = k\xi\eta \sin 2\varphi, \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where

$$k = \frac{1}{\sqrt{r_1 r_2}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The two planes passing respectively through the axis Oz and the lines $O\xi$ and $O\eta$ are the directrix planes R_1 and R_2 of the hypar. From eq. (3) it can be seen that all planes parallel to R_1 cut the surface in straight lines, likewise all planes parallel to R_2. The hyperbolic paraboloid thus contains two systems of straight lines (rulings). The lines of each system all run parallel to the corresponding directrix plane, but their slope varies with their distance from that plane (see *fig. 2a*).

If we consider a part of the surface of a hypar shell, bounded by two straight lines AB and CD of the one set of rulings and two straight lines AC and BD of the other set (see fig. 2*b*), we see at once that a hyperbolic paraboloid is also obtained when a straight line (e.g. AC), which intersects two skew straight lines (AB and CD), slides along the latter two lines, while remaining parallel to a given plane (in this case the directrix plane to which AC and BD are parallel).

Describing now a parallelogram on two adjoining lines (e.g. AB and AC) of the above figure, the fourth corner of the parallelogram being E, the line DE will run parallel to the axis of the hyperbolic paraboloid and is termed the *linear distortion* v of the hypar surface $ABDC$.

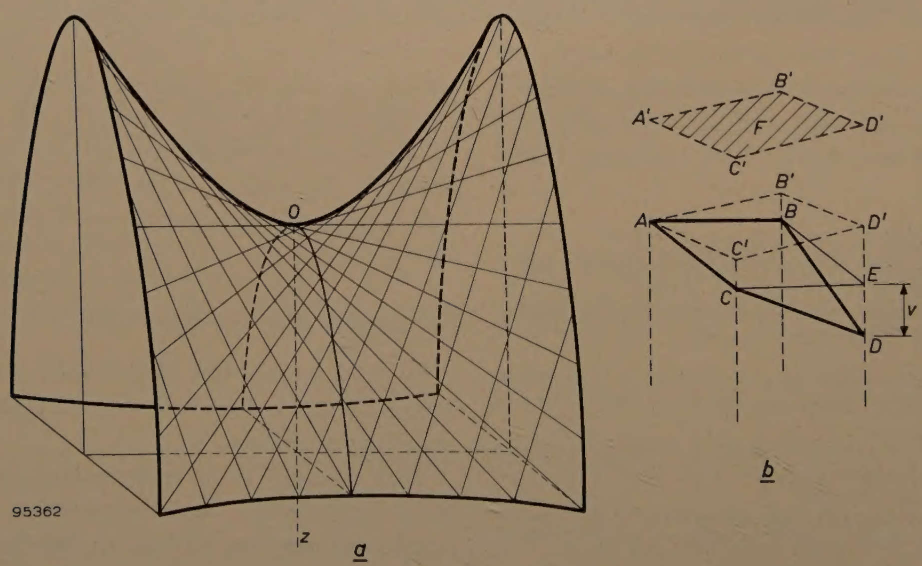


Fig. 2. *a*) Hyperbolic paraboloid showing the two sets of ruling lines. O = apex, Oz = axis. *b*) Part of a hyperbolic paraboloid bounded by four rulings AB, CD, AC, BD. The "linear distortion" is v, the specific distortion is $v/F = k = 1/\sqrt{r_1 r_2}$.

If we project the surface $ABDC$ on to a plane perpendicular to the axial direction DE, and if the area of this projection (parallelogram $A'B'D'C'$) is F, we call the ratio v/F the specific distortion, this being identical with k of formula (4) and constant for all parts of the hypar shell, bounded by four ruling lines. In practice the quantity k is usually determined by calculating the specific distortion.

Finally, the hyperbolic paraboloid can also be regarded as a translation surface. For this purpose let us revert to fig. 1. All planes parallel to the plane of symmetry xOz cut the surface, according to eq. (1), in parabolae which are congruent with p_1, while all planes parallel to the plane of symmetry yOz give intersecting curves congruent with the parabola p_2. We can therefore also imagine the hyperbolic paraboloid as produced by displacing parabola p_2 parallel to itself, its apex gliding along p_1, or by the parallel displacement of parabola p_1, its apex gliding along p_2.

The plane $z = +c$ intersects the hypar, according to eq. (1), in a hyperbola. Projecting this onto the plane xOy, the lines $O\xi$ and $O\eta$ are the asymptotes of this hyperbola, and Ox and Oy its real and imaginary axes respectively. For the plane $z = -c$, the projected intersecting curve is again a hyperbola, again with $O\xi$ and $O\eta$ as asymptotes, but with Ox as its imaginary axis and Oy as its real axis.

In the special case that $\varphi = 45°$ ($O\xi$ and $O\eta$ then being perpendicular to each other) the above intersecting curves are rectangular hyperbolae and the surface is then called a rectangular hyperbolic paraboloid.

If we take an arbitrary hypar and project a number of contour hyperbolae and a number of ruling lines onto the plane xOy, we obtain a diagram such as that of *fig. 3*.

For the sake of completeness we should mention the one-shell hyperboloid, which also contains two systems of ruling lines. These lines, however, are no longer parallel to two directrix planes, but parallel to the generators of a conic surface, the latter being the asymptotic cone of the hyperboloid. A consequence of this fact is that it is much more difficult to calculate the distribution of forces in a hyperboloidal shell than in a hyperbolic-paraboloidal shell.

The membrane theory of the hypar shell

Forces lying in the median surface of a plate or shell are known as membrane forces. (The median surface is defined as the locus of the mid-points of the thickness everywhere in the plate or shell.) Unlike a flat plate, a curved shell is able to take a load perpendicular to its surface in the form of membrane forces. When a shell is allowed to deform freely under a given set of forces, a good approximation to the actual distribution of forces is obtained by assuming that exclusively membrane forces are acting. This is plausible since for a statically possible stress distribution in which no bending or torsion takes place (i.e. a quasi-two-dimensional stress distribution) the deformation energy of a shell structure is approximately a minimum. The usual procedure, therefore, is to begin by calculating the distribution of forces in the hypar shell in accordance with the membrane theory, and subsequently to apply corrections to allow for stress disturbances at the edges of the shell, where the deformations resulting from the membrane stresses are *not* able to take place freely. In the Philips pavilion, for example, at the intersection of each pair of hyperbolic-paraboloidal surfaces, the surfaces are rigidly joined by a rib. The condition here is that the deformations of the rib must be the same as those of the abutting edges of the shells. This gives rise to stresses near the edges, so-called edge disturbances, which we shall consider presently.

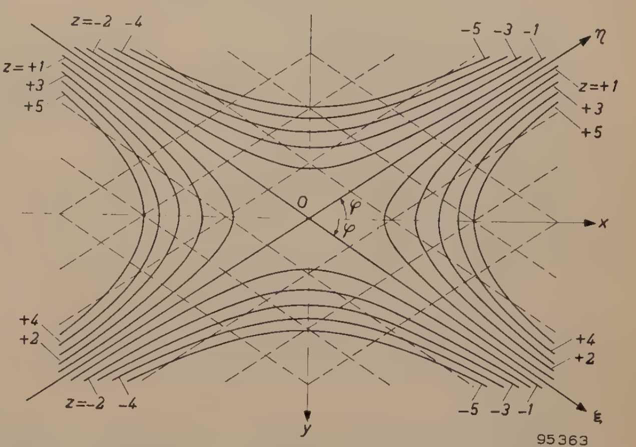


Fig. 3. Projection on the plane xOy of contour lines (hyperbolae) and ruling lines (dashed) of a hyperbolic paraboloid.

The differential equations for the membrane state of stress can best be derived by considering the equilibrium in the directions ξ, η and z of a loaded element. For this purpose we consider a small element of the shell bounded by four neighbouring ruling lines which, when projected on to the horizontal plane $\xi O\eta$, form an elementary parallelogram of sides $d\xi$ and $d\eta$ (see *fig. 4*). The components of the applied load per unit projected horizontal area of the shell in the directions $O\xi$, $O\eta$ and Oz are denoted p_ξ, p_η and p_z respectively. The oblique membrane forces per unit length (called shell forces, analogous to stresses in the more general case of solid bodies) in the shell element are denoted n_ξ, n_η and ϑ.

The projected shell-forces are:

$$\left.\begin{aligned} \bar{n}_\xi &= n_\xi \frac{\cos\alpha}{\cos\beta}, \\ \bar{n}_\eta &= n_\eta \frac{\cos\beta}{\cos\alpha}, \\ \bar{\vartheta} &= \vartheta . \end{aligned}\right\} \quad \ldots\ldots \quad (5)$$

The equilibrium conditions in the ξ, η and z directions yield the following equations:

$$\left.\begin{aligned} &\frac{\partial\bar{n}_\xi}{\partial\xi} + \frac{\partial\vartheta}{\partial\eta} + p_\xi \sin 2\varphi = 0, \\ &\frac{\partial\bar{n}_\eta}{\partial\eta} + \frac{\partial\vartheta}{\partial\xi} + p_\eta \sin 2\varphi = 0, \\ &2\vartheta \frac{\partial^2 z}{\partial\xi\,\partial\eta} + \left(p_z - p_\xi \frac{\partial z}{\partial\xi} - p_\eta \frac{\partial z}{\partial\eta}\right) \sin 2\varphi = 0 . \end{aligned}\right\} \quad (6)$$

The expression between brackets is the z-component $\bar{p}_z$ of the net load at the point of the shell under consideration when the latter is resolved into two components, one ($\bar{p}_z$) in the z direction and the other in the tangent plane; the third equation of (6) can therefore also be written:

$$2\vartheta \frac{\partial^2 z}{\partial\xi\,\partial\eta} + \bar{p}_z \sin 2\varphi = 0 . \quad \ldots \quad (7)$$

A very simple solution is found when the applied load in the z direction per unit horizontal area of the hypar shell is everywhere constant ($= \bar{g}$, say), while $p_\xi = p_\eta = 0$. From eq. (3) we derive the purely geometrical relation

$$\frac{\partial^2 z}{\partial\xi\,\partial\eta} = k \sin 2\varphi .$$

For the case $\bar{p}_z = \bar{g}$, eq. (7) thus gives

$$\vartheta = -\frac{\bar{g}}{2k} = \text{constant.} \quad \ldots\ldots \quad (8)$$

We see from this formula that the shell force ϑ is inversely proportional to k. In connection with (4) it is therefore advantageous to make the radii of curvature r_1 and r_2 as small as possible. The more pronounced is the curvature of the shell, the more favourable is the stress distribution.

If the shell is bounded by ruling lines, and if the membrane forces n_ξ and n_η at the boundaries may be assumed to be zero (non-rigid edge members), it follows from the first two equations of (6) that these forces are zero at all points of the shell.

A load uniformly distributed per unit horizontal area is therefore transmitted by a hypar shell of vertical axis and constant thickness to the edge members in the form of constant shear stresses along the rulings. The shell is then for practical purposes a structure of *equal strength*, which means that if at one point of the shell the stress reaches its maximum permissible value, it will do so at all points. Since in that case the strength of the material is fully exploited everywhere, it is evident that a structure of equal strength requires a minimum of material.

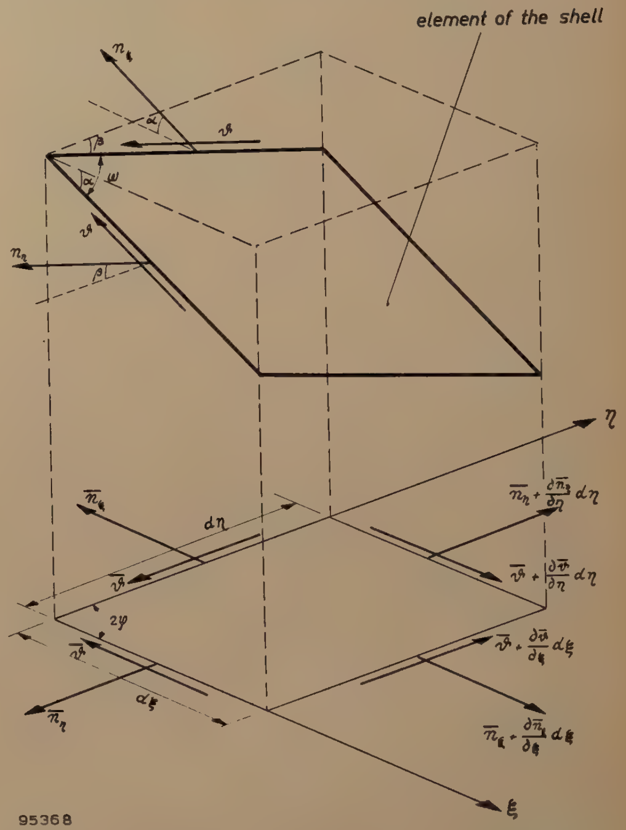


Fig. 4. Equilibrium of a shell element, introducing the oblique shell-forces projected onto the $\xi\eta$ plane. Shell-forces are the forces per unit length of the shell; the components in the median plane of the membrane are generally denoted by n and ϑ. A shell-force divided by the thickness of the shell evidently represents a stress (force per unit area).

The above is further illustrated in *fig.* 5, which depicts a simple shell consisting of four quadrants, each constituting a part of a rectangular hyperbolic paraboloid with edge members. As indicated in one of the quadrants by dashed lines, there are two sets of parabolae on the surface, a set with the convex side upwards and another set with the concave side upwards. The former are in compression and the latter in tension. The load on the shell is now borne for one half by the compression parabolae and for the other half by the tension parabolae. If we consider a point on an edge member where a tension and a compression parabola meet, we see that the reactions of both together produce a shear force along the edge member, so that the latter

are not loaded perpendicularly to their axes. The same manner of force-transmission also takes place in non-rectangular hypar shells. This explains why the load in the case of the Philips pavilion is transmitted largely in the form of compressive forces along the ribs to the foundations and why the vertical stanchions originally envisaged for the support of the ribs could eventually be dispensed with.

The distribution of forces as described here applies only to a load uniformly distributed per unit horizontal area, as for example a load of snow of constant (vertical) thickness.

For the dead weight of the shell, the simple distribution of forces holds only approximately. If this weight be g per unit area of the shell, then the shell-force for a shell with a vertical axis (fig. 1) is:

$$\vartheta = -\frac{g}{2k}\sqrt{\Phi}, \quad \ldots \ldots \quad (9)$$

where

$$\Phi = 1 + k^2(\xi^2 + \eta^2 - 2\xi\eta\cos 2\varphi). \quad . \quad (10)$$

If the normals to the shell surface do not make large angles with the axis ($\leqslant 15°$), we can put Φ equal to unity. Furthermore we find:

$$\left.\begin{aligned}\bar{n}_\xi &= -\frac{g}{2k}\cos 2\varphi\sqrt{\Phi} + \\ &+ \tfrac{1}{2}g\eta\sin^2 2\varphi \ln\left[\sqrt{\Phi} + k\xi - k\eta\cos 2\varphi\right] + f_1(\eta), \\ \bar{n}_\eta &= -\frac{g}{2k}\cos 2\varphi\sqrt{\Phi} + \\ &+ \tfrac{1}{2}g\xi\sin^2 2\varphi \ln\left[\sqrt{\Phi} + k\eta - k\xi\cos 2\varphi\right] + f_2(\xi).\end{aligned}\right\} \quad \ldots \ldots \quad (11)$$

The integration functions $f_1(\eta)$ and $f_2(\xi)$ must be determined from the boundary conditions.

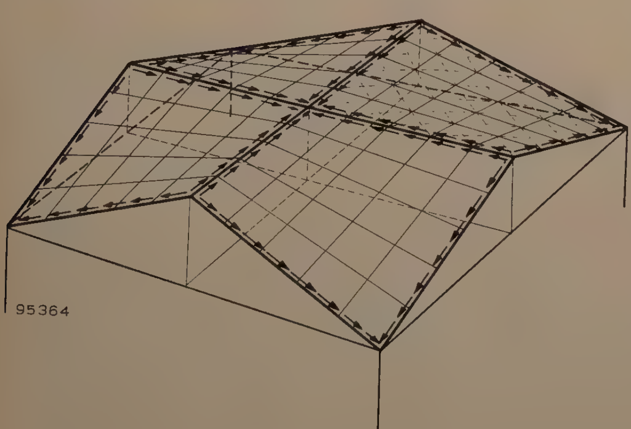


Fig. 5. System of four rectangular hypar shells, loaded uniformly per unit horizontal area. The arrows indicate the shear forces transmitted to the rigid edge members.

In this way we can also calculate the membrane forces for a constant load w per unit area of shell directed everywhere normally to the shell. Such a load is often assumed to represent a *wind* load. We then find:

$$\vartheta = -\frac{w\Phi}{2k}, \quad \ldots \ldots \quad (12)$$

$$\left.\begin{aligned}\bar{n}_\xi &= wk(2\xi\eta - \xi^2\cos 2\varphi) + f_3(\eta), \\ \bar{n}_\eta &= wk(2\xi\eta - \eta^2\cos 2\varphi) + f_4(\xi).\end{aligned}\right\} \quad . \quad . \quad (13)$$

From the above formulae we see that, as opposed to a snow load, the shell-forces ϑ are no longer constant under a dead weight or wind load, and that the shell forces n now begin to enter into account.

Having calculated the projected shell-forces $\bar{n}_\xi$ and $\bar{n}_\eta$, the actual shell-forces n_ξ and n_η are also known, from equations (5). It should be noted here that (see fig. 4):

$$\left.\begin{aligned}\cos\alpha &= \frac{1}{\sqrt{1 + (\partial z/\partial\xi)^2}}, \\ \cos\beta &= \frac{1}{\sqrt{1 + (\partial z/\partial\eta)^2}}.\end{aligned}\right\} \quad . \quad . \quad . \quad (14)$$

Finally, in order to judge the strength of the structure, we must determine from the actual shell-forces the *principal shell-forces* [6]) in magnitude and direction for a number of characteristic points of the shell. For this purpose we must know the angle ω between the ruling lines (see fig. 4) at the point under consideration. This is given by the formula:

$$\cos\omega = \frac{(\partial z/\partial\xi)(\partial z/\partial\eta) + \cos 2\varphi}{\sqrt{[1 + (\partial z/\partial\xi)^2]\,[1 + (\partial z/\partial\eta)^2]}}. \quad (15)$$

Since we are concerned here with ***oblique*** shell-forces, or stresses, it is necessary to modify somewhat the conventional *Mohr circle* construction. ***Fig. 6a*** shows the conventional construction, for a plane state of stress; such a diagram enables us to determine graphically the direction and magnitude of the principal stresses σ_1 and σ_2, from the normal stresses σ_x and σ_y and the shear stress τ, acting in two mutually perpendicular plane elements (both perpendicular to the stress-free plane). The modified construction is shown in fig. 6*b*; this enables us to determine the direction and magnitude of the principal shell-forces n_1 and n_2 from the oblique shell-forces n_ξ, n_η and ϑ, acting in two plane elements

[6]) Defined analogously to the principal stresses in the case of a plane state of stress, i.e. as the shell-forces in those planes in which there are no shear forces but only normal forces (these are, incidentally, the largest and smallest shell-forces at the point in question).

which make an acute angle ω with each other [7]).

Further, one can construct in the median surface of the shell two sets of curves such that the tangents at each point of intersection represent the directions of the principal shell-forces n_1 and n_2 in that point. These curves are then the principal shell-force "trajectories"; these are sometimes called the principal *stress* trajectories, since the shell stresses are simply the shell-forces divided by the shell thickness.

If prestressed concrete is to be used, one of the questions arising concerns the minimum tensile forces that should be applied in the cables (laid in the direction of the ruling lines); in other words, what are the minimum compressive shell-forces that must be superimposed in the ξ and η directions on the existing stress distribution in order to prevent the occurrence of a tensile stress in any plane element through the point concerned? The graphical solution of this problem is shown in fig. 6*c*.

[7]) C. G. J. Vreedenburgh, Hyperbolic Paraboloidal Shells, collected and edited by W. Grijm, Central Ctte., Studiebelangen, Delft 1954, pp. 17-26.

Edge disturbances

As remarked in the foregoing, the shell in the proximity of a stiff edge cannot freely undergo the deformations arising from the membrane state of

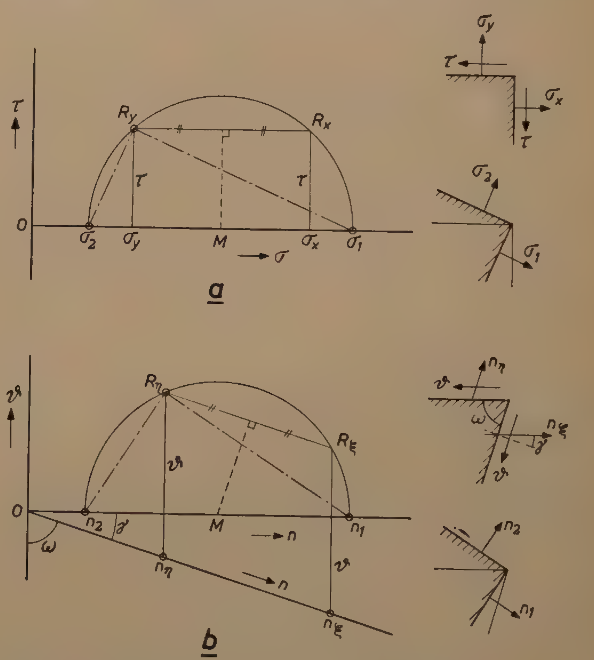
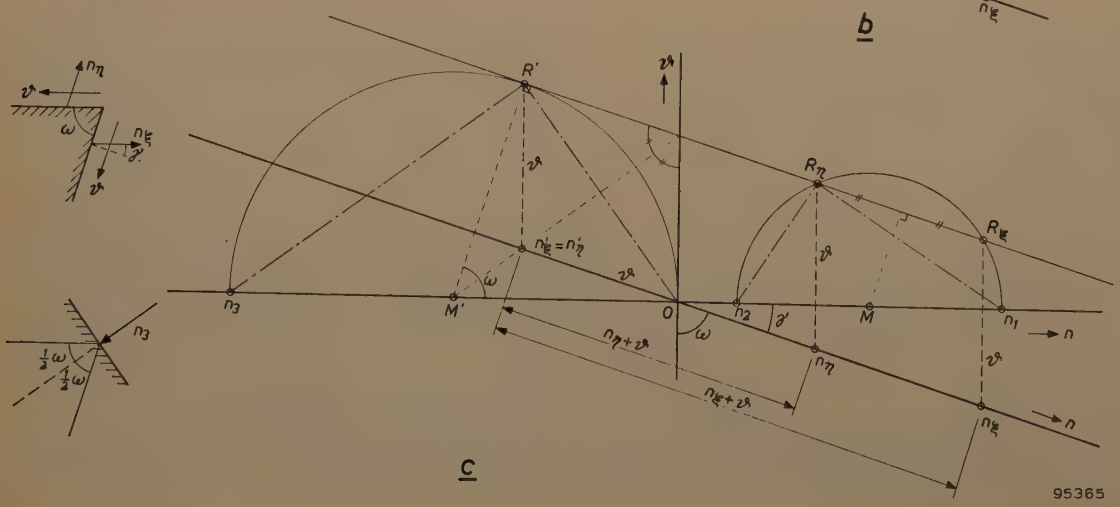


Fig. 6. *a*) Mohr's construction for a plane state of stress. The construction gives the principal stresses σ_1 and σ_2 at any given point when the normal stresses σ_x and σ_y and the shear stress τ in two mutually perpendicular plane elements at that point are known. By plotting σ_x and σ_y on the horizontal axis and τ on the vertical axis, we find the points R_x and R_y. The circle through R_x and R_y, with the centre M on the σ axis, yields the points σ_1 and σ_2. The distances $O\sigma_1$ and $O\sigma_2$ give the magnitude of the principal stresses, their directions being given by $R_y\sigma_1$ and $R_y\sigma_2$, respectively.

b) Modification of Mohr's construction, adapted for the stresses (or, since a membrane state of stress is involved, the shell forces n_ξ and n_η, see fig. 4) in the direction of the ruling lines passing through a point of the hypar. The plane elements in which these forces act are *not* mutually perpendicular, but make an angle ω with each other. Plotting n_ξ and n_η on an axis set at an angle ω with the ϑ axis, and drawing two lines of length ϑ through these points, parallel to the ϑ axis, yields R_ξ and R_η. The circle through R_ξ and R_η with centre M on the relevant n axis yields the points n_1 and n_2. The distances On_1 and On_2 give the magnitude of the principal shell-forces, acting respectively in the directions $R_\eta n_1$ and $R_\eta n_2$.

c) In order to obtain a state of stress entirely free of tensile stresses, Mohr's stress circle must be displaced by external forces in such a way that it comes to lie entirely to the left of the ϑ axis. This can be achieved by means of prestressing cables, applied in the direction of the ruling lines of the hypar; the prestressing shell-forces (compressive forces), directed along n_ξ and n_η, must then be set out from the points n_ξ and n_η on the same oblique axis towards the left.

The *minimum* prestressing is obtained when the circle is tangent to the ϑ axis and when at the same time the new points R_ξ' and R_η', which lie on the same oblique line as R_ξ and R_η, are made to coincide; the circle is then also tangent to this oblique line (at point R'). The centre M' of the circle is therefore found on the line bisecting the angle between the oblique line and the ϑ axis. The figure shows that the prestressing must cause the compressive shell-forces $n_\xi + \vartheta$ and $n_\eta + \vartheta$ (giving the point $n_\xi' = n_\eta'$) and that the state of stress is then converted into a linear compressive stress with a principal shell force On_3 (the other principal stress being zero); On_3 acts in the direction $R'n_3$, i.e. in the direction of the abovementioned bisector.



stress. If we imagine for a moment the edge member to be separated from the shell, so that the deformations can take place unhindered, it is clear that the edge of the shell and the edge member will no longer be a precise fit. A fit is only possible when the edge member exerts forces and moments on the edge of the shell (normal, shear and transverse forces, and bending and torsional moments) and the shell exerts opposite forces and moments on the edge member such that the extra deformations enable a perfect fit to be obtained. The calculation of these edge disturbances, which must be superimposed on the membrane state of stress, is one of the most difficult problems of shell theory.

If we approximate to the hyperbolic-paraboloidal shell in a small region by a translation surface of circles of radii r_1 and r_2, then, for small curvatures, the following (tetra-harmonic) differential equation for the edge disturbances is approximately valid:

$$\nabla^8 w = -\frac{D}{K}(1-\nu^2)\left[\frac{1}{r_2^2}\frac{\partial^4 w}{\partial x^4} - \frac{2}{r_1 r_2}\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{1}{r_1^2}\frac{\partial^4 w}{\partial y^4}\right], \quad (16)$$

where

w = displacement of a shell point in the direction of the normal,

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}, \quad D = \frac{E\delta}{1-\nu^2}, \quad K = \frac{E\delta^3}{12(1-\nu^2)},$$

δ = shell thickness,
E = modulus of elasticity and
ν = Poisson's ratio.

The quantities D and K represent the tensile and bending stiffness respectively of the shell.

If w is known, we can then find the entire distribution of forces. We find that the disturbances originating from the edge points always consist of the superposition of two spatially periodic waveforms; in many cases both waveforms are rapidly "damped", so that at some distance from the edge members little of the edge disturbance is perceptible.

Partly because a calculation based on equation (16) is extremely complicated, and indeed, not feasible in the case of such intricate boundary conditions as exist in the Philips pavilion, the following approximate calculation of the order of magnitude of the edge disturbances in hypar shells seems to be entirely adequate for practical purposes. The method adopted is based on the fact that, so far as bending phenomena are concerned, a shell can be compared with a plate supported on an elastic foundation. Thus if we imagine a strip of the hypar shell perpendicular to an edge member, this strip will behave to a first approximation as a beam supported on an elastic foundation. If the local principal curvatures [8]) of the hypar shell at the edge point in question be k_1 and k_2, then the coefficient of reaction (foundation modulus) for the equivalent beam on an elastic foundation is approximately:

$$c = E\delta(k_1^2+k_2^2). \quad \ldots \ldots \quad (17)$$

(The coefficient of reaction of an elastic foundation is the reaction per unit area when the deflection is equal to unity. The larger c, the greater the rigidity of the foundation.)

It is now possible by simple means to calculate the variation of the edge disturbance, which is in this case determined solely by the bending moment m and the transverse force q, both per unit length of the shell. It is found that the behaviour of the edge disturbances can be described in terms of a "wavelength" and a "damping", both of which are determined by a characteristic length:

$$\lambda = \frac{0.76\sqrt{\delta}}{\sqrt[4]{k_1^2+k_2^2}}. \quad \ldots \ldots \quad (18)$$

At a distance of about 3.5 λ from the edge, the edge disturbance can be assumed to be negligible. From (18) we see that the zones of appreciable edge disturbance are smaller the thinner the shell and the larger the principal curvatures (i.e. the smaller the principal radii of curvature).

If the load on the shell perpendicular to its surface is equal to p, and if we assume that the shell is clamped perfectly rigidly to the edge member, the edge-disturbance moment m and the edge-disturbance shear force q, both per unit length of shell, will vary with the distance x from the edge as illustrated in *fig. 7*. From the formulae given in the figures it can be seen that each edge-disturbance has the form of a single damped "wave", which can be regarded as the resultant of the two waveforms which satisfy equation (16). The damping is stronger the smaller the characteristic length λ. In the case considered, assuming perfect clamping at the edge, the negative clamping shell-moment is:

$$m_0 = \tfrac{1}{2}p\lambda^2, \quad \ldots \ldots \quad (19)$$

and the support reaction per unit length (= the shell shear force at the clamping position) is:

$$q_0 = p\lambda. \quad \ldots \ldots \quad (20)$$

If the shell is hinged to the edge, the support reaction becomes:

$$q_0 = \tfrac{1}{2}p\lambda, \quad \ldots \ldots \quad (21)$$

[8]) In a hyperbolic paraboloid the principal directions of curvature at a given point coincide with the bisectors of the acute angle formed by the two rulings passing through that point, and its supplement.

and the maximum (positive) shell-moment, occurring at a distance of 0.785λ from the edge member, is:

$$m = 0.16\, p\lambda^2. \quad \ldots\ldots \quad (22)$$

If the edge member itself can undergo deformation, the influence of this deformation can also, if necessary, be taken into account.

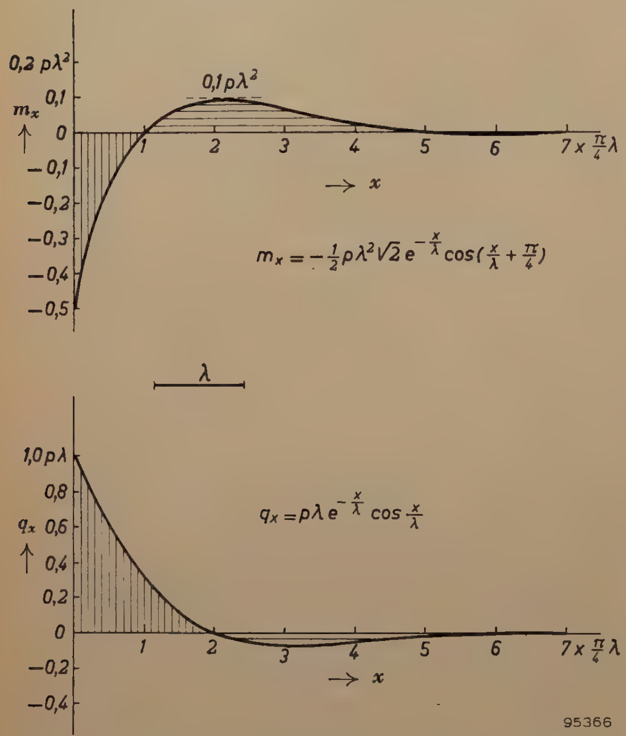


Fig. 7. Curves showing the edge disturbances m (bending moment per unit length of shell) and q (transverse force per unit length of shell) as a function of the distance x from the edge, when the shell is clamped in a perfectly rigid edge member. λ is the characteristic length. At a distance $x = 3.5\lambda = 4.5\ (\pi/4)\lambda$ from the edge, the edge-disturbance waveform is practically damped out.

With the aid of formulae (19) and (22) it is possible to determine the order of magnitude of the edge-disturbance moments for any arbitrary shell. If these moments become unduly large, the shell must be strengthened at the relevant position. In the case of reinforced concrete shells it is sufficient in many cases to add extra steel. Otherwise the shell will have to be made thicker.

With formulae (20) and (21) we can calculate what fraction of the total shell load is transmitted to the edge members by *bending*. Evidently the remaining part is borne by the shell in the form of membrane forces.

The formulae given here, and in particular formula (8), formed the basis for the theoretical determination of the distribution of forces to be expected in the Philips pavilion, and for the provisional dimensioning of the ribs and shell walls, also for the purposes of the model tests.

It may be noted here in passing that the usefulness of formula (19) was recently demonstrated experimentally in the Stevin laboratory at Delft, in the course of tests on a large reinforced concrete model of a hyperbolic-paraboloidal shell as in fig. 5.

Stability against buckling and second-order buckling

It is known that doubly-curved shells are much more stable against buckling than cylindrical ones. For estimating the buckling load p_k of a hyperbolic-paraboloidal shell, i.e. the load perpendicular to the shell surface under which the shell is about to buckle, we can apply Wansleben's theory [9]). We then find:

$$p_k = \frac{2E\delta^2}{\sqrt{3(1-\nu^2)}}\, k_1 k_2, \quad \ldots \quad (23)$$

where k_1 and k_2 respresent the absolute values of the principal curvatures at the point in question.

When $k_1 = k_2$, eq. (23) transforms into Zoelly's formula for a spherical shell. For a more rigorous calculation of the buckling load of a rectangular hyperbolic-paraboloidal shell the reader may be referred to a paper by Ralston [10]).

From formula (23) it can be seen that the buckling load increases, that is to say the danger of buckling decreases, proportionately as the curvatures increase. In the case of the Philips pavilion it was therefore primarily necessary to concentrate on the stability of those parts where the curvatures were very slight. As appears from formula (23), however, the danger of a shell buckling can be substantially reduced by increasing the thickness of the shell. In any case, in applying this formula, a large safety factor must be introduced to allow also for the possible presence of errors in the shape of the shell.

It is not an easy matter to determine theoretically the stability of shells, particularly when, apart from buckling phenomena, second-order buckling ("oil-canning") is to be taken into account. The latter phenomenon consists of the shell suddenly adopting a new position of equilibrium, involving displacements of finite magnitude. The conventiona- theory, which assumes only infinitely small deformal tions, is no longer applicable, and must be replaced by a second-order theory. As regards the danger of second-order buckling, which is more serious than the danger of buckling, it seems to us that the

[9]) K. Girkmann, Flächentragwerke, Springer Vienna, 4th ed., 1956, pp. 516-529.

[10]) A. Ralston, On the problem of buckling of a hyperbolic paraboloidal shell loaded by its own weight, J. Math. Phys. 35, 53-59, 1956.

hyperbolic-paraboloidal shell will be less vulnerable, owing to the saddle form, than the spherical shell. Although this has not yet been proved theoretically, our conjecture is nevertheless confirmed to some extent by the very high stability of hypar shells, as observed in the model tests on the Philips pavilion which are described in the third article in this series.

III. MODEL TESTS FOR PROVING THE CONSTRUCTION OF THE PAVILION

by A. L. BOUMA *) and F. K. LIGTENBERG **).

624.023.744:061.41(493.2):725.91

When the contracting firm "Strabed" approached Professor Vreedenburgh at the end of January 1957 for advice on the building of Philips pavilion, the first question was whether the design of Le Corbusier and Xenakis was indeed realizable as a shell structure of reinforced concrete. The plan of the contractors "Strabed" was to use concrete, 5 cm thick, for making the walls, which were designed as hyperbolic paraboloids, these shell walls to be reinforced in their lines of intersection by cylindrical ribs 40 cm thick.

In the previous article of this series it was explained that the stresses in hyperbolic-paraboloidal ("hypar") shells can readily be calculated if a membrane state of stress can be assumed. This will only be the case, however, if certain boundary conditions are satisfied. In a structure of such intricate shape as the pavilion under discussion, these boundary conditions are *not* fulfilled in many of the shells, giving rise to much more complex states of stress which are scarcely amenable to exact mathematical analysis. To answer the above question, the obvious procedure was therefore to perform tests on scale models, and to introduce calculations merely for supplementing the experimental results.

Fig. 1. Framework of tubes and wire-gauze for making the plaster model of Philips pavilion, scale 1 : 25 (T.N.O. Institute, Rijswijk, Netherlands).

For this purpose the Netherlands Institute T.N.O. for Building Materials and Building Constructions prepared a model, scale 1 : 25, consisting of plaster of Paris on a wire-gauze framework (*fig. 1*). Owing to the term fixed for the decision by Messrs Strabed, little more than a week was available for carrying out this investigation, and the results could not

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therefore be considered as any more than a guide. The tests led to the conclusion that, with some minor modifications, the design was feasible.

After Philips had awarded the contract for building the pavilion to Strabed, the investigation was continued more extensively. The object was to get some idea of the stresses occurring in the ribs and walls, in order to determine exactly certain dimensions and the amount of prestressing needed, particularly since the architect had meanwhile altered his design in some respects (see I). The most important of these alterations were also introduced in the model. Furthermore, an attempt was made, by applying overloads, to assess the stability of the structure. In this investigation the structure was regarded as being seamless (monolithic).

In fact the contractors Strabed did not intend to build the pavilion as a monolithic structure. They planned to form the shell surfaces from precast slabs which, after all the surfaces had been built up, were to be prestressed by means of high-tensile steel wires disposed on the inside surfaces of the building. Naturally this method of construction introduced additional problems, and it was therefore desirable to investigate these problems, and in particular the effect of the prestressing, on a new model. To this end a 1:10 scale model was built in the Stevin Laboratory of the Department of Civil Engineering at the Delft Technische Hogeschool. The model, consisting of two adjoining shell surfaces, omitted all details of the design but otherwise simulated as realistically as possible the proposed method of construction (*fig. 2*).

Fig. 2. The 1 : 10 scale model for investigating the system of construction planned by Messrs Strabed (Stevin laboratory, Technische Hogeschool, Delft).

Model tests, general remarks [1])

If model tests are to faithfully simulate the behaviour of a structure subjected to an increasing load up to the moment of collapse, the model must be a perfect scaled-down facsimile of the original and be made of the same material. Phenomena such as crack formation, yielding and rupture will then occur at places analogous to where they would occur in reality and under stresses identical with those associated with the corresponding phenomena in the actual structure.

Since a stress has the dimensions of load per unit area (kg/cm^2), the loads per unit area of the model must be the same as those for the actual structure. If the scale of the model is $1:n$, the following rules are applicable:

Stresses (kg/cm^2)	$1/1$
Forces per unit length (kg/cm)	$1/n$
Forces (kg)	$1/n^2$
Moments per unit length (kg)	$1/n^2$
Strains (specific deformations, $\varepsilon = \Delta l/l = \sigma/E$)	$1/1$

It is by no means always possible, however, to make a model of the same material as the actual

[1]) See J. B. Wilbur and C. H. Norris, Structural model analysis, Chapter 15 in Handbook of experimental stress analysis, edited by M. Hetényi, Wiley, New York 1950.

structure. In the case of the Philips pavilion, for example, it was not practicable to make the 2 mm thick model walls of concrete. One must look then around for another material which is suitable. That presents no particular difficulties if one wishes to investigate the behaviour of the structure merely in the elastic region: one can measure strains (specific deformations) and determine from these the stresses occurring. In that case it is sufficient if the material of the model is elastic within an adequately large range of deformations and satisfies Hooke's law. The material will not, generally speaking, possess the same modulus of elasticity as that of the actual structure. The stresses being equal, the specific deformation in the model is inversely proportional to the modulus of elasticity.

If only the *stress distribution* is of interest, it is no longer necessary to adhere to the rule that the stresses in the model should be equal to those in reality. Stresses can be chosen that are readily measurable on the model material, and the latter (which may, for example, be steel) may then in fact be better suited to experimental investigation than the actual material (e.g. concrete). If the load per unit area of the model is larger, for example, than it will be in reality, the local stresses measured in the model are everywhere also proportionately larger than those that will occur in practice.

The *buckling load* is proportional to the modulus of elasticity of the material. This means that the buckling load of the actual structure is equal to the ratio of the respective moduli of elasticity times the load at which the model begins to show signs of buckling.

In some cases it is also possible to depart from the actual structure by adopting different *relative dimensions* for certain parts of the model, thereby simplifying the building of the model. One must then know, however, which parameters are important for the behaviour of the structure (such as for instance the area of the cross-section or the second moment of area), and for these the true values must be maintained.

A further complication arises if either the material of the actual structure or the model material is not *homogeneous*. This was the case with the plaster model, and we shall go into this in more detail.

Tests on the plaster model

As already mentioned, the first model was made of plaster of Paris on a framework of wire-gauze, the scale being 1 : 25 (fig. 1). The average thickness of the walls was 2.1 mm. The ribs (edge members) were steel tubes with a diameter averaging 8.4 mm and a thickness of 0.7 mm. The tubes were slightly thickened by plaster.

The tensile strength of plaster of Paris is rather higher than that of concrete, and as a material it obeys Hooke's law fairly well. The finished model, a structure of plaster and steel, could therefore be regarded as behaving elastically and in accordance with Hooke's law, so that the *strains* measured on this model could be interpreted with fair accuracy as *stresses*.

For applying the loads a large loading frame was built, by means of which the forces exerted by weights were conveyed via lever systems to the surfaces of the model (*fig. 3*). Measurements were made with the model subjected to:

a) loading by its own weight;

b) a vertical load on a single, fairly horizontal surface, to investigate the effect of a snow load;

c) wind pressure on the surfaces separately, enabling the effect of wind pressure on the windward side and wind suction on the lee-side to be ascertained for two wind directions.

The most unfavourable combined loads of deadweight with snow or wind produced the extreme stresses, moments and forces in the structure.

For measuring horizontal and vertical displacements, altogether 40 displacement dial-gauges, accurate to within 0.01 mm, were placed at various positions on the ribs. The strains were measured with strain gauges, a total of 40 being used for the first tentative tests, and 130 for the subsequent detailed investigation (*fig. 4*). In the latter investigation four strain gauges per measuring point were fixed to the surfaces, two on the outside, often at right-angles to each other, and two parallel to these on the inside. In this way it was possible to determine the force and the moment to which the cross-section concerned was subjected. Two, and where possible three, strain gauges per measuring point were fixed to the ribs.

Since the material of the model was not homogeneous (wire-gauze and plaster), it was not possible to deduce directly the stresses in the actual construction by multiplying the measured strains by the modulus of elasticity of the model material. The following procedure was therefore adopted for interpreting the strain measurements.

Suppose that at a certain point of the model wall the strains ε_1 and ε_2 were measured on the inside and outside surfaces. In the case of a *homogeneous* material, with modulus of elasticity E, the corresponding stresses are then:

$$\sigma_1 = E\varepsilon_1 \quad \text{and} \quad \sigma_2 = E\varepsilon_2$$

Fig. 3. Plaster model with the lever system for simulating wind loads.

Fig. 4. Equipment used for measurements on the plaster model. A series of displacement dial-gauges are mounted on angle struts and connected via long rods to the model (shown more clearly in fig. 3 above). A large number of strain gauges are fixed to the shell surfaces and the ribs for determining the stresses (whence the elaborate wiring). Some of the indicating instruments are visible in the foreground.

(the influence of the transverse contraction being here neglected). The cross-section, of thickness t, must now transmit per unit length a normal force

$$N = \frac{\sigma_1 + \sigma_2}{2} t ,$$

and, introducing I, the second moment of area, a bending moment

$$M = (\sigma_1 - \sigma_2)\, I/t .$$

Introducing the *tensile stiffness* $D = Et$ and the *bending* (*flexural*) *stiffness* $K = EI$ of the cross-section, we may write:

$$\left. \begin{aligned} N &= D\frac{\varepsilon_1 + \varepsilon_2}{2}, \\ M &= K\frac{\varepsilon_1 - \varepsilon_2}{t}. \end{aligned} \right\} \quad \ldots\ldots \quad (1)$$

These relations now apply equally well to a *non-homogeneous* material. Since all the data were not available for calculating the tensile stiffness D and the bending stiffness K of the non-homogeneous material used, these quantities were determined for the model by combining a calculation with a number of simple measurements. For this purpose several test plates of the same construction as the walls of the model were subjected to a purely tensile load and to a bending load. The values found were $D = 40\,000$ kg/cm and $K = 70$ kg cm, the wall thickness being $t = 2.1$ mm. From the strains measured on the model, the forces and the moments per unit length, N and M, were calculated with the aid of formulae (1), and by applying the model rules it was then possible to find N and M in the actual structure (N being n times, and M being n^2 times larger than in the model).

An entirely analogous procedure was adopted for the ribs. From the measured strains the strain ε_z was determined in the centre of gravity of a cross-section. The normal force in the cross-section is then:

$$P = B\varepsilon_z ,$$

where B is the tensile stiffness. This was again found experimentally on a number of test rods. According to the model rules the normal force P in the actual structure is n^2 times larger.

Once the tensile stiffness, for example, has been determined for the wall material, one can find with the aid of the formula $D = E't$ an "equivalent modulus of elasticity" E' for an imaginary, equally thick but homogeneous wall with the same D. The value found was $E' = 2 \times 10^5$ kg/cm², and analogously, by representing the bending stiffness by $K = E''I$, E'' was found to be 10^5 kg/cm². These moduli of elasticity are thus lower than those for the actual structure of concrete, for which E can be put as 3×10^5 kg/cm². The same can also be done for the ribs. With the aid of these equivalent moduli of elasticity the measured strains can also be directly interpreted as stresses in the case of the ribs.

The test programme was carried out completely both on the original design and on the modified design. The original envisaged a number of stanchions for supporting the three peaks. The forces in these stanchions were found to be generally slight. In the modified design it was assumed that the function of these stanchions could safely be taken over by the neighbouring oblique ribs. It was in fact found that the omission of the stanchions had very little influence on the stresses in the model, and therefore this modification of the design was accepted.

The stresses in the walls were found on the whole to be by no means negligible, sometimes exceeding 20 kg/cm² and in one instance reaching a maximum of 40 kg/cm². This applied particularly to the large vertical surfaces. Johansen's yield line theory offers a reliable method of determining the ultimate load of transversely-loaded flat plates [2]). This theory was accordingly applied to several very flat wall sections to gain some idea of their collapse load.

The extreme values of the forces in the ribs (both tensile and compressive) were found to be about 30 tons, while the extreme values of the stresses — partly caused by bending — amounted to −60 kg/cm² and +80 kg/cm².

After the question as to the feasibility of the structure had been answered by the preliminary measurements, Strabed was able to determine broadly the required amount of prestressing on the basis of the stresses found in the surfaces and ribs.

The interpretation of the measured strains naturally involved various uncertainties. Owing to variations in the thickness of walls and ribs it was impossible to be certain that the actual stress, or force or moment, had been found for all cases. Moreover, as far as the walls were concerned, it was not known exactly where and in what direction the maximum force or the maximum moment occurred,

[2]) R. Hognestad, Yield line theory for the ultimate flexural strength of reinforced concrete slabs, J. Amer. Concrete Inst., March 1953.
P. Lebelle, Calculs "à rupture" des hourdis et plaques en béton armé, Ann. Inst. Tech. Bâtiments et Trav. publ., No. 85, Jan. 1955.

and as regards the ribs it was not known at what points the extreme values of the stresses appeared. The measurements were therefore made at more or less arbitrary points, and it was impossible to check whether the greatest stresses had in fact been observed. The distribution of forces will also have been influenced somewhat by the fact that the ratios of the tensile stiffness and bending stiffness in the walls and ribs of the model were not entirely in accordance with those in reality. Nevertheless the results gave reason to assume that the measurements carried out at the points chosen provided a reasonable idea of the distribution of forces arising under dead-weight, and with wind and snow loads.

The Belgian building authorities had laid down that the building must be able to withstand loading by its own weight plus a double wind load (150 kg/m²) with complete safety. The model was accordingly subjected to this load, and finally to a load of 1.5 times its own weight and a slightly higher wind load. Even under these conditions no particular effects were noted. In view of the fact that the buckling load is proportional to the modulus of elasticity, this meant that a monolithic structure of a homogeneous material obeying Hooke's law and possessing a modulus of elasticity equal to that of concrete (at least 300 000 kg/cm²) would be able to withstand a load almost 3 times higher than the above loads. It must be said at once, however, that concrete is not homogeneous and elastic, nor does it obey Hooke's law. Phenomena such as crack formation, plastic yield and creep in concrete can appreciably reduce the safe strength of the building.

The interpretation of the above results on the plaster model in relation to a concrete structure thus left some matters open to doubt. The formation of cracks, however, can be prevented by prestressing. Moreover the system of construction plays an important part, as will be shown in the following.

Investigation of the system of construction on a plywood model

The investigation described in the foregoing was based on the assumption that the Philips pavilion was a monolithic shell structure, the stiffness of the surfaces being approximately identical in all parts. It was the intention of Strabed, however, to build up the shell surfaces from precast slabs, which were first to be erected at the site on scaffolding and then pressed together by prestressing wires. Additional problems were created by these plans, the more so because it was intended to arrange the prestressing wires on the inside of the building only.

The theory was that the prestressing force, although apparently eccentrically applied by the wires inside the building, would nevertheless be effectively exerted centrally in the shell surfaces. The prestressing wires exert on a rib the forces V_1 and V_2, which can be combined to form a resultant R (*fig. 5a*). This resultant R can be resolved

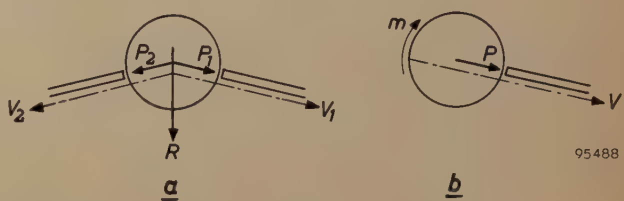

Fig. 5. Forces in a rib, viewed in cross-section.
a) Rib with two abutting shells. By means of prestressing wires, fixed to the inside surface of each shell, forces V_1 and V_2 are exerted on the rib. The resultant R of these forces can be resolved into almost centrally exerted compressive forces P_1 and P_2, by which the shells are prestressed.
b) Rib with only one abutting shell. Here, too, a compressive force P can be made to act centrally on the shell by introducing, besides the force V of the prestressing wires, a torsional prestressing of moment m, in the rib.

into forces P_1 and P_2 which the rib exerts on the shell surfaces abutting on it. As appears from fig. 5*a*, R can be so resolved as to cause the compressive forces P_1 and P_2 to appear practically centrally in the wall surfaces. By allowing a certain freedom in the magnitude of the prestressing forces it is also possible to adjust the orientation of P_1 and P_2.

If only one shell surface abuts on a rib, *torsional prestressing* can be introduced into the rib (see article IV) to produce a moment m of opposite sign, so that the prestressing force V in this case too is effectively exerted as a central compressive force P in the shell surface (see fig. 5*b*). Such a torsional prestressing may, of course, also be usefully applied when two shell surfaces abut on the rib.

Where the shell surfaces meet the ground, a special substructure, which is later rigidly fixed to the foundation beam, can be arranged to supply a moment, whereby here too the prestressing force V is exerted as a central compressive force P in the shell surface.

A structure when prestressed will undergo deformations. If these deformations are opposed by, say, a relatively stiff or fixed section, it becomes a particularly complicated matter to ascertain the effect of the prestressing. A large proportion of the prestressing forces will be lost in this stiff or fixed section and will not be exerted where they are required. For this reason it was thought necessary

to arrange that the substructure could move freely during the prestressing of the shell surfaces.

It was decided to investigate the effect of the movable substructure during the prestressing of the shell on a separate model. The model could also be used to ascertain whether the prestressing in the shell surface did in fact act centrally as postulated above, and whether the pattern of stresses set up in the walls corresponded to that of the stresses in the prestressing wires on the surface of the wall. If the latter was not everywhere the case, there would be a risk of buckling in those parts where the internal (compressive) force per unit length in the cross-section was appreciably larger than the external (tensile) force.

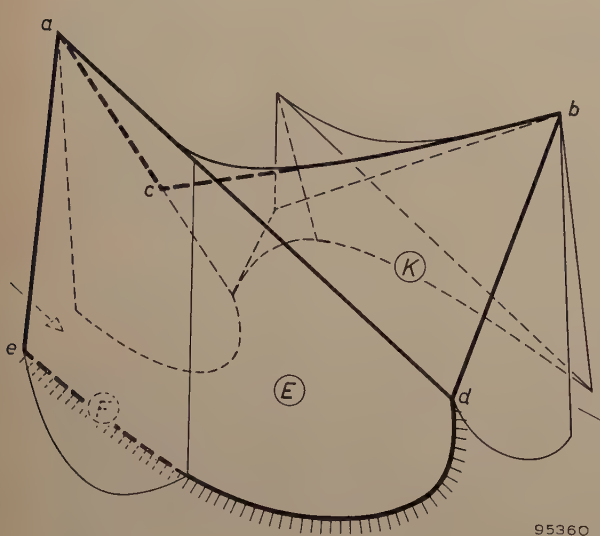

Fig. 6. Sketch of the Philips pavilion, indicating the surfaces E and K from which the plywood model was built (cf. fig. 18 in I). E is bounded by the straight lines *ae* and *ad* and the hyperbola *ed*; K is bounded by the twisted "quadrangle" *acbd*. The model was supported by vertical stanchions under points *a*, *b* and *c*.

From the plaster model a reasonable idea had already been gained as to the behaviour of the structure as a whole. Since the above-mentioned problems concerned details connected with the system of construction, it was sufficient to build a model of only a *part* of the Philips pavilion. The areas selected were those marked E and K in *fig. 6* (see also fig. 18 in I). All details, such as the windows permitting a view into the equipment room, the interior floors and canopies, and in particular the exterior shell surface F meeting E almost in the middle, were omitted. It was expected that all the shell surfaces in the actual Philips pavilion would be subject to more favourable conditions than the two surfaces investigated, which contained *inter alia* a very high, almost entirely flat section; hence

Fig. 7. The plywood model in course of construction. In a manner similar to that to be used in the actual erection, the slabs are fitted together on wire "scaffolding", the wires being spanned between the previously erected ribs in accordance with rulings of the hyperbolic-paraboloidal shells.

any difficulties attaching to the system of construction should come to light readily in the investigation of these two surfaces.

To simulate closely the system of construction the model was built on a large scale (1 : 10). The shell sections were made of plywood slabs (3-ply), each of which was bent, by means of a rigid attachment, to the appropriate saddle shape and finished to ensure an accurate fit (*figs. 7, 8* and *9*). Each slab had an area of approximately 100 cm², so that many hundreds of slabs, and a great deal of work, were needed to build a model 2 m high and 3 m long. A realistic prestressing system was created fairly simply by using prestretched nylon thread for the prestressing wires.

Since the modulus of elasticity of the plywood used was 165 000 kg/cm², i.e. about half that of concrete, the prestressing and the loads were kept

Fig. 8. Detail of a wall of the plywood model, illustrating the bending of each plywood slab to fit a certain part of a saddle surface. In the centre of the photograph is a photo-elastic measuring element.

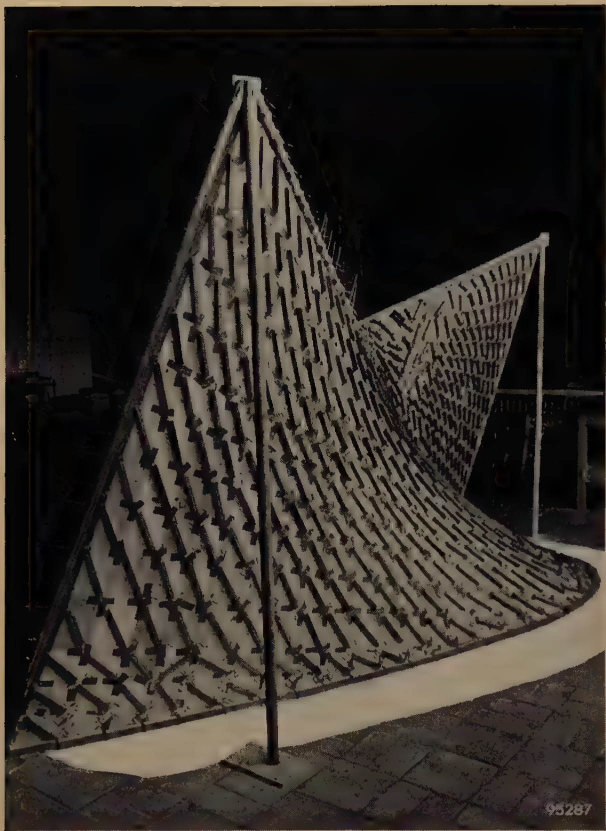

Fig. 9. The complete plywood model.

to half their actual values. Under these conditions, according to the model rules discussed at the beginning of this article, the occurrence of buckling phenomena and rupture (in the joints) in the model should correspond to the manner of their occurrence in the actual structure. The deformations were reproduced on a scale of about 1 : 10.

It was evident that, even without many measurements, a model of this kind would provide considerable insight into the behaviour of the structure during the process of prestressing. Consequently, fewer measurements were carried out than on the plaster model, the emphasis being laid on visual observation of how the model behaved under diverse circumstances.

Fig. 10. Prestressing the plywood model.

The plywood slabs from which the model was built up were temporarily held together by strips of adhesive tape (fig. 8). This structure was then uniformly prestressed (*fig. 10*). At first it was found that those ribs of the model which were free on one side were eccentrically twisted by the prestressing, and buckling occurred in a part of the steep upright surface. To prevent this happening it was necessary, as described above with reference to fig. 5*b*, to exert torsional moments on these ribs equal in magnitude to those which would result from the prestressing of the abutting surface (not present in the model). These torsional moments were produced by fixing a large number of spokes at right angles to the ribs, (see also fig. 9) and by arranging for forces to act on the ends of these spokes in directions perpendicular to both the ribs and the spokes. The forces were exerted by wires passing over pulleys and to which weights were suspended (*fig. 11*). This improvement having been introduced, the prestressing was able to be applied without difficulties.

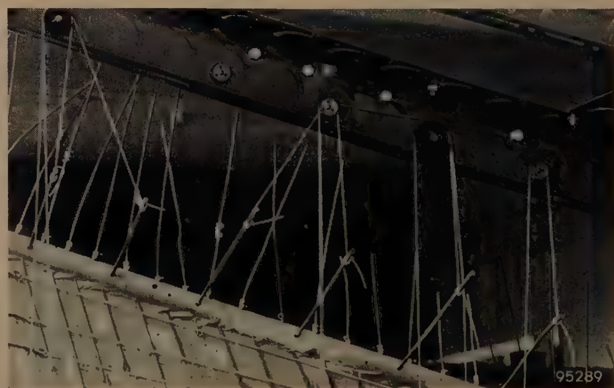

Fig. 11. In the free ribs (to which no second hyperbolic-paraboloidal surface abuts in the model), spokes are fitted by means of which moments can be applied to the ribs (see fig. 5*b*) to oppose the torsional moment exerted by the prestressing wires. This is done by exerting transverse forces on the spokes by wires passing over pulleys and from which sandbags are suspended (see also fig. 10).

The result of this test was entirely satisfactory. The effect of the prestressing was indeed found to be as expected, and the movable substructure also came up to expectations.

Although the model dit not exactly simulate the actual structure as far as the action of forces due to loading are concerned, it was nevertheless thought worth while to investigate the effect of subjecting the model to a load corresponding to its own weight plus wind. Loading due to dead-weight was hitherto

absent, because the weight of the plywood slabs amounted to only about 1/25 of the dead-weight per unit area of the actual structure, whereas it was necessary, owing to the lower modulus of elasticity of the model, to apply *half* the actual dead-weight per unit area. This was done by slowly and uniformly applying increasing numbers of sandbags to the two wall surfaces (*fig. 12*).

When the load had almost reached its full value (corresponding to 0.9 of the dead-weight), the lower surface E suddenly began to buckle along an almost horizontal joint in the flat upright section of the wall close to the ground. This buckling was quite unexpected. Originally it was thought that the almost horizontal section of the same surface would be much more dangerous and for this reason the steep part of the wall had been built up much more simply in the model than it was to be in the actual pavilion. In particular, as stated above, the exterior wall surface F abutting at this position had been omitted (see fig. 6). It was not, therefore, necessarily correct to conclude from this buckling that the actual pavilion would not be able to carry its own weight, though the appearance of such a phenomenon was to some extent a warning signal.

Following this phenomenon the load was removed from the model. Not until the load had been removed in the upper surface K did the model return to its original state, from which it was concluded that the reaction forces in the surface supporting this surface had been in part responsible for the phenomenon. Owing to the absence of the abutting surfaces, the upper surface K will have acted more as a thick plate than as a shell, that is to say the state of stress comprised not solely forces and reactions lying in the surface itself (membrane state of stress) but also bending moments and reactions perpendicular to the plane.

Fig. 12. The plywood model during the application of a load corresponding to the dead-weight of the structure.

The area that had buckled was subsequently stiffened with two ribs in order, among other things, to simulate the effect of the adjoining surface F; this produced a stiffness more in accordance with that of the actual structure. It was now found that, when loaded by its own weight and even by very heavy wind loads on both sides, the structure was well able to stand up to all the forces applied (*fig. 13*).

Fig. 13. Arrangement for applying a wind load (tensile load from sandbags hung over pulleys).

Finally, the model was overloaded by allowing several persons to climb on to it (up to about 1.5 × the dead-weight). Although this represented extremely heavy loading, the model was still capable of withstanding shocks and wind loads, so that it may be said to have passed this test very satisfactorily (*fig. 14*).

This result, then, promised well for the actual pavilion, although the original buckling when the model was loaded by its own weight suggested possible dangers. In this respect, however, the jointing of the slabs is of essential importance, and this was impossible to simulate entirely realistically in the model.

Fig. 14. The plywood model severely overloaded.

It seemed desirable to carry out additional tests in order to investigate more closely the buckling stability of a shell surface composed of slabs. This could not be done, however, owing to lack of time. After consultation with Le Corbusier, Philips decided that it would be preferable to apply prestressing wires also on the outside of the building. It seemed to us that this would sufficiently strengthen the coherence of the surfaces and the erection of the Philips pavilion could therefore be started with full confidence in the outcome.

IV. CONSTRUCTION OF THE PAVILION IN PRESTRESSED CONCRETE

by H. C. DUYSTER *). **624.023.744:061.41(493.2):725.91**

When the Philips concern asked us to submit a proposal on how the pavilion designed by Le Corbusier and Xenakis might be built, our thoughts turned at once in the direction of self-supporting concrete shells, which would be no thicker than the minimum of 5 cm specified for reasons of soundproofing by the acoustic engineers. In the design submitted to us (see article I, figs. 15-19) the walls were composed of hyperbolic paraboloids, or of surfaces that could readily be transformed into hyperbolic paraboloids, and from experience gained in various projects as well as from theoretical considerations (in part quite new) [1]), we were aware of the outstanding qualities of strength and stability possessed by shells formed as hyperbolic paraboloids ("hypar" shells). Even without detailed analysis we were able to arrive at an idea of the forces and deformations that would occur in the structure as designed, so that we were in a position with the help of elementary calculations, to estimate the strength that would be required at the various points.

We shall not go into the subsequent developments — the more exact calculations by Professor Vreedenburgh, the modification of the first design by Xenakis, the structural tests carried out on scale models, etc. — all these having been sufficiently treated in the preceding articles in this issue (I, II and III). We shall confine ourselves here to questions concerned with the construction of the building in prestressed concrete [2]).

Why prestressing?

It will perhaps be useful first of all to recapitulate briefly the idea underlying the prestressing of concrete. Concrete can safely withstand very considerable compressive stresses — up to 150 kg/cm^2 in the case of good quality concrete (i.e. made of good materials and carefully manufactured). It is much less capable, however, of withstanding tensile stresses. In order to use concrete in structures where tensile stresses will occur, one of two courses can be adopted: 1) The concrete can be *reinforced*, the reinforcement consisting of steel rods around which the concrete is cast, leading to firm internal adhesion: these rods take up the tension (*fig. 1*). 2) The concrete can be *prestressed*, by means of steel wires which are fixed to both ends of a concrete constructional element; when the wires are tensioned a compressive stress is produced in the concrete, and with appropriate dimensioning the resultant stresses even when the element is loaded will everywhere be *compressive* stresses (*fig. 2*).

Having decided on concrete as the material eminently suitable for giving shape to the ideas embodied by Le Corbusier and Xenakis in their design, we were thus left with the choice between reinforced and prestressed concrete.

Now, for shells as thin as these, reinforced concrete is not an attractive proposition. To make good, homogeneous concrete in the form of such thin layers and for such large and often steep walls as in the present structure, is in itself extremely difficult. When reinforcement is present the difficulties become well-nigh prohibitive. There are thus objections to the *principle* of reinforced concrete. With prestressed concrete, on the other hand, there are no such inherent difficulties: the prestressing wires can be applied not inside the concrete but on its surface, if desired. In fact, the *shape* of the walls of the Philips pavilion lent itself exceptionally well to this treatment owing to the fact that hyperbolic paraboloids may be generated by straight lines, a property possessed in common with ruled surfaces in general; this made it possible to apply all or most of the prestressing wires such that they would run *straight*. This would make it easier to introduce the appropriate prestresses in the material.

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*) "Strabed": Société de Travaux en Béton et Dragages, 11 rue Gineste, Brussels.

1) See article II, and the literature there referred to.

2) The most important details of the work have already been described: see H. C. Duyster, Cement **9**, 447-450, 1957 (No. 11-12).

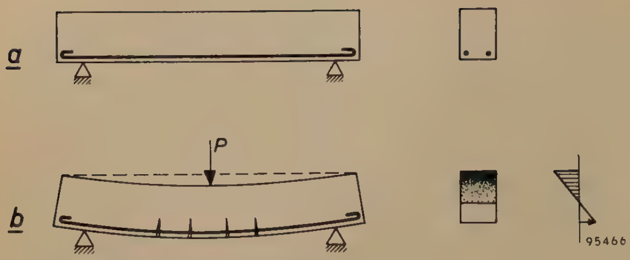


Fig. 1. Principle of reinforced concrete.
a) Beam with steel reinforcement, in unloaded state (dead-weight disregarded).
b) Beam subjected to a load *P*. The stippling in the transverse section indicates the occurrence of an axial compressive stress in the concrete. Hair cracks appear in the concrete at the lower surface of the beam, and the steel reinforcement comes under tensile stress. On the extreme right is shown the vertical distribution of the axial stresses (compression towards the left, tension — in the reinforcement — towards the right).

Though the choice necessarily fell on prestressed concrete for the practical reason mentioned, it is important to note that prestressed concrete is, in fact, an intrinsically better solution of the problem than reinforced concrete. The reasons for this are as follows (see also article II).

In a hyperbolic-paraboloidal shell, a load uniformly distributed over its surface (parallel to the hypar axis) is transmitted to the edge members in such a way that, to a first approximation, normal forces alone arise in these members. The shell is then left with purely membrane stresses, which is the most favourable state from the standpoint of strength. The edge members carry the load as a normal force down to the points of support on the foundation.

In reality the picture is not quite as simple as this, differences being introduced mainly as a result of deformations in the structure. Although

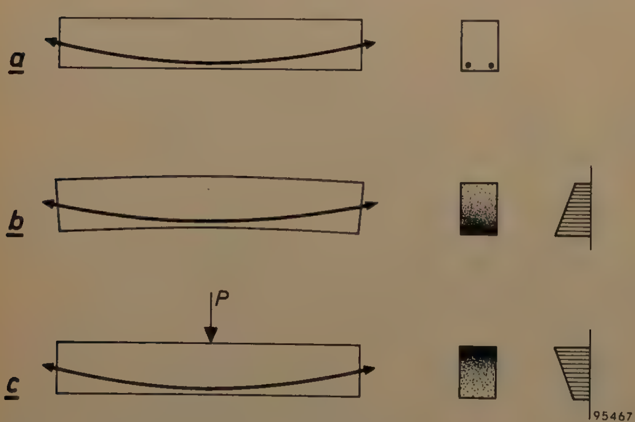


Fig. 2. Principle of prestressed concrete.
a) Beam with prestressing wires, in unloaded state. The wires run through parabolic channels through the concrete.
b) Beam after prestressing, still unloaded. The beam is deformed; compressive stresses appear in the concrete, distributed as shown by the stippling in the transverse section and the graph on the right.
c) Prestressed beam under a load *P*. The stress distribution changes but nowhere do tensile stresses occur in the concrete.

primarily the shell ought to be able to transmit the load solely by membrane stresses, deformation gives rise to secondary (bending) stresses, and the trajectories of the principal stresses are distorted. These effects are also discussed at greater length in article II. The flatter the shell the more the stress-distribution differs from that of a membrane state of stress. Usually a very localized but pronounced deviation is found near the edge members, for the deformations of the latter are of a different order of magnitude and vary along the edges otherwise than in the thin shell itself.

By appropriate prestressing of both the shell and the edge members it is possible to limit the disparities in deformation that are responsible for the occurrence of often relatively large secondary stresses. It will be more economical, then, to construct the shell in prestressed concrete than in reinforced concrete, for the prestressed shell can take up the load mainly with membrane stresses and hence approaches a true "shell" construction, whereas in a shell of reinforced concrete the secondary stresses produced by deformation will generally predominate: it may therefore require much greater wall thicknesses and possibly even additional strengthening with supports.

Two other advantages of prestressing must be mentioned:

a) Of practical, though not vital importance is the fact that, owing to the elimination of tensile stresses in the concrete, the hairline cracks often occurring in concrete structures are entirely avoided. As a result, less stringent demands can be made on the waterproofing of the shells. This is particularly convenient, since to provide the unusually-shaped shells with conventional roofing would be a highly difficult if not impracticable operation.

b) A most essential feature of prestressing, and perhaps the *sine qua non* for the building of the Philips pavilion, is that it enabled us to form the shell surfaces from *precast slabs*. We shall deal with this in more detail.

Construction of the hypar shells from precast concrete slabs

It would have been very difficult, even without reinforcement, to cast the thin and, in many cases, strongly twisted shells of the pavilion *in situ* by means of wooden shuttering. The most experienced workers in concrete would scarcely have been equal to the task, especially when such high demands are made on the quality of the concrete work. Even before "Strabed" was awarded the contract,

Fig. 3. Concrete slabs being cast on a sand-bed to form a part of a hyperbolic-paraboloidal shell. The mound of sand, containing a certain proportion of loam, is levelled off with a plank sliding over two edges, after the limiting rulings of the shell section have been defined by four planks. The sand is coated with a thin skin of cement. By positioning slats of wood 1 cm thick along the ruling lines the entire surface of the section is divided into lozenge-shaped moulds, of approximately 1 m², into which the concrete is cast to produce slabs 5 cm thick (a light reinforcement mesh is introduced to prevent breakage during transport to the site). The checker-board structure visible on the photograph is due to the system of first casting the "black" squares, the slats being prevented from giving by rods driven into the sand; subsequently, when this concrete is set, the rods are removed and the "white" squares are cast.

we had therefore devised the plan to form each hypar shell from sections which could be cast in open sand-bed moulds. This system of "prefabrication" also offered the advantage that it could be carried out under cover and hence independent of weather conditions; for this purpose a shed stood at our disposal some kilometres from the building site. The extent to which each shell had to be subdivided depended in the first place on the available height in the shed and on the permissible steepness of the slopes of the sand bed (shuttering would again have been necessary if the slopes were too steep). What was even more important was that the cast sections should be easy to handle and transport. With this in mind the size of the precast slabs was fixed at about 1 m². One sand bed was formed for each shell section consisting of a few dozen slabs.

At the site the slabs were fitted together in a manner which will be described, and the joints filled with mortar (the operation thus being a rather odd kind of brick-laying process).

This slab-wise construction of each shell was, of course, only possible owing to subsequent prestressing of the entire structure: the bond between adjacent slabs is even less able to withstand tension than the concrete itself, but thanks to appropriate prestressing no tensile stresses can ever occur anywhere in the walls. The effect of prestressing is to make the entire structure, including the ribs, behave as if it had been cast as one whole.

The prefabrication process brings out even more clearly than the considerations in the previous section how remarkably well the hyperbolic-paraboloidal shell is adapted to construction in prestressed concrete. The sand beds on which the lozenge-shaped concrete slabs were to be cast — each slab individually moulded as part of a particular hyperbolic paraboloid — were very easily made by delineating them, following the ruling lines, with straight planks of wood, and then levelling them off (*fig. 3*). Particular care was needed, however, in setting out the ruling lines along the edges and the curved lower boundary of a shell, as well as in carrying over the corresponding angles from one section of the shell to the next. This would naturally have been simpler if we had been able to make *all* slabs of each shell on one sand bed, but this was not possible for the reasons given (surface steepness, shed height). Following the directions of the ruling lines, slats of wood 1 cm thick were fitted on the sand bed and the slabs cast between them. With a view to transport from the shed to the site, the slabs were provided with a light reinforcement mesh.

We must return at this point to the remarks made at the beginning of this article concerning the choice between reinforced and prestressed concrete. The artifice of prefabricating the hyperbolic-paraboloidal shells in small elements without shuttering brings reinforced concrete into the picture again — as we have just seen — as a reinforcement mesh to facilitate transport. It might seem, then, that in spite of what was said earlier, the shell walls could be made of reinforced

Fig. 4. The 40 cm thick cylindrical ribs were cast *in situ* in "shuttering" erected on scaffolding.

concrete. The reinforcement of the slabs is of no help, however, for *forming* the large walls, for there is no possibility whatsoever of joining up the reinforcement of adjacent precast slabs in such a way that it would keep the concrete free from tensile stresses with the whole structure loaded.

The edge members fitted at the intersections of each pair of hyperbolic-paraboloidal shells are cylindrical concrete ribs 40 cm in diameter. They were cast *in situ* in shuttering on a framework of wooden struts and stays (*fig.* 4) while the more than 2000 slabs were being cast in the shed. It was rational to make the ribs cylindrical, because the shells "turn" about the ribs; with any other shape it would have been troublesome to join the shells to the ribs. Nor would there have been any sense in providing the ribs with any but a circular cross-section, for their purpose was after all to take substantially normal forces and therefore no particular flexural stiffness was required of them.

After the ribs had been cast, further wooden scaffolding was erected, of which the outer-surface beams followed the ruling lines of the hyperbolic-

Fig. 5. The precast slabs, only 5 cm thick, were placed in position and mortared on scaffolding incorporating wooden beams along the ruling lines of the shells.

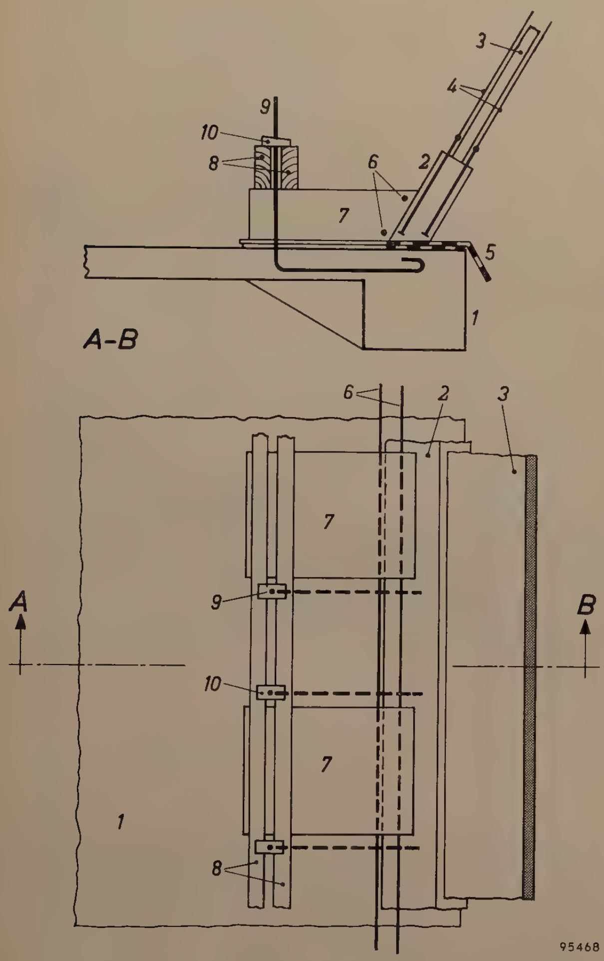
paraboloidal surfaces. The slabs were then placed in position on the scaffolding and provisionally held in place (*fig. 5*). The 1 cm wide joints between the slabs having been filled, the entire structure was prestressed, after which the scaffolding was removed and the finishing touches put to the shell surfaces.

Closer consideration of the prestressing of the shells and ribs

The requisite prestresses in the shells were derived from the results of the model tests performed in the T.N.O. Institute (see III, sub-section entitled "Tests on the plaster model"). The loads allowed for were the dead-weight of the structure together with the sound-proofing, a snow load, and a wind load of 75 kg/m² from various directions.

The necessary prestresses were to be introduced in the shells by systems of 7 mm steel wires laid over the surface and post-tensioned, each wire being capable of taking a maximum tensile force of about 3300 kg. Assuming that the forces so introduced in a shell would everywhere act centrally (i.e. in the median plane of the shell) and would follow the direction of the wires, it was possible to write down the formulae giving the principal stresses produced at each point by the applied forces, as well as the stresses produced along the joints between the slabs and perpendicular thereto. In this way we could ascertain the number and position of the prestressing wires needed.

The assumption just mentioned seems justified in the case of hyperbolic-paraboloidal shells, where the prestressing wires can follow straight lines, always provided, however, that two conditions are satisfied, viz. the wires must be disposed with reasonable uniformity over the shell surface, and the shell must be capable of freely undergoing deformation during the application of the prestresses. The latter condition called for special measures at



a

b

Fig. 6. *a*) In order for the prestressing to be completely effective, the bottom edge of a shell must be able to move a few cm (without canting, i.e. changing its inclination) over the foundation, during the prestressing process. For this purpose two layers of bituminous felt *5* are introduced between the foundation *1* and the substructure *2* against which the shell *3* is pulled tight by prestressing wires *4*. The substructure itself is prestressed by wires *6* running along the perimeter. Canting is prevented by tail blocks *7* cast on to the edge beam and anchored to the foundation. Anchorage is effected by means of temporary wooden beams *8*, which are fixed by clamps *10* to flexible rods *9* cast into the foundation between the tail blocks.
b) Substructure with tail blocks. In order to fix the whole structure to the foundation, the space between the tail blocks is filled with concrete after the shells have been erected and prestressed (hence after the deformation has taken place). The reinforcement for this concrete filling (not shown in fig. *a*) can be seen between the blocks.
It may be pointed out that the tail blocks were in all cases *inside* the building. In the part of the building shown here, the wall bends outwards. It may also be pointed out that as the substructure can slide only horizontally and any rotational movement is completely prevented, any eccentrically-acting prestress is made to act centrally in the shell: the bending moment will be taken up by the foundation, leaving none in the shell (see III, fig. 5*b*).

various points, particularly as regards the shells resting on the foundation; the substructure of these shells was initially separated from the foundation by two layers of bituminous felt (which also served later for waterproofing the building from the surrounding water channel); this permitted relative movement between substructure and foundation until after prestressing was completed, when the whole structure was joined to the foundation by the addition of concrete (*fig. 6*). Without this measure some of the prestressing forces would have been taken up by the foundations and the stresses introduced in the shell would certainly not have had the right direction. The tests carried out on the plywood model, described in III, showed that this construction functioned as required and, in general, that the above assumption was justified in our case: the fact that no buckling occurred at any part of the model (modified as described, p. 25) during prestressing provided the assurance that the transmitted forces substantially followed the direction of the wires.

In accordance with the original plan the wires in these prestressing tests were applied to only one side of the shell surfaces (corresponding to the inside of the building). Further tests were needed to ascertain whether the shell surfaces would have sufficient stability against buckling when subjected to external loads, but owing to lack of time it was decided with Le Corbusier's approval to apply prestressing wires to the outside surfaces as well (see *fig. 7*). This simplified the problem appreciably.

The wires mainly followed the ruling lines of the hyperbolic-paraboloidal surfaces, but in the case of some shells whose rulings intersected at very sharp angles it was necessary also to apply wires that followed the bisectors of the obtuse angle between the ruling lines. These wires formed the exception in that they followed a curved path.

The ribs were also prestressed, in three distinct ways. Firstly, *compressive* prestresses had to be produced by straight wires so that the ribs could take up the tensile stresses measured in the model tests. Secondly, some ribs had to be given a *bending* prestress, by means of wires following a parabolic path in a plane through the axis (cf. fig. 2). This was necessary in particular in the case of two ribs at the entrance and exit, to each of which only one almost flat shell abuts. Thirdly, *torsional* prestresses were introduced in a number of ribs to counteract

Fig. 7. The pavilion shortly before completion, showing the numerous prestressing wires of high-tensile steel which are applied to both surfaces of the shells. Each wire is 7 mm thick and is tensioned with a force of 3300 kg.

the torsional moments imparted to them by the prestressed abutting shells. Although the concrete of the ribs was capable of taking up these torsional moments, it was desirable to eliminate them to ensure centralized prestresses in the shells (see III, fig. 5*b*). As far as we know, this is the first application of torsional prestressing in concrete.

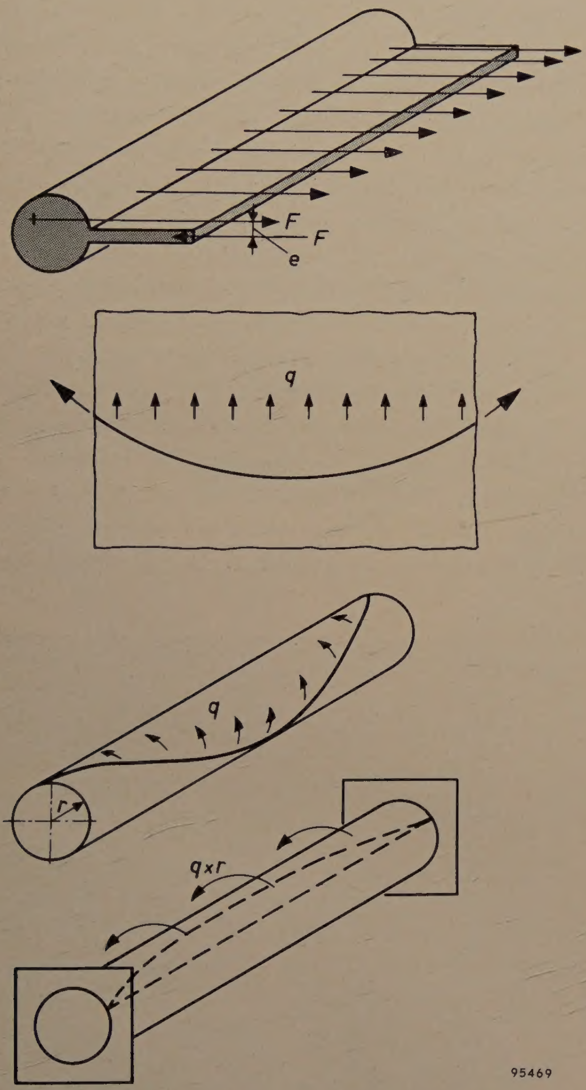


Fig. 8. At places where the prestressing forces (F kg/m, i.e. kg per linear meter) do not act centrally through the shell, (*a*), the rib is loaded by a torsional moment ($F \times e$ kg.m/m), which give rise to bending moments in the shell. These can be eliminated by introducing a torsional prestressing in the rib. For this purpose it is necessary to tension wires along a path that would be parabolic on the surface of the rib. This may be explained as follows. Consider first a plane on which a parabolically curved wire is fixed as shown in (*b*). When the wire is tensioned, the plane is loaded by a uniformly distributed force q kg/m. If this imaginary plane is now wrapped around a cylinder (*c*), the latter is loaded by a uniformly distributed torsional moment $q \times r$ kg.m/m when the wire is tensioned. When q is given the proper value, this moment will exactly compensate for the distortion caused by a uniformly distributed torsional moment $F \times e$ (cf. *a* and *d*). (The parts of the structure to which the ends of the beam are connected and in which the prestressing wire is anchored, e.g. the foundation and the junction with other ribs, are naturally subjected in this process to a torsional load, owing to the reactions to the tensile forces of the wire.)

Fig. 9. Prestressing wires for a rib, mounted before making the shuttering and casting the concrete. This particular rib was fitted only with wires for producing torsional prestressing (eight wires running on the circumference of a cylinder, 35 cm in diameter, formed by a series of iron hoops). The 9 mm wires look thicker here since they have been coated with a preparation to prevent bonding with the concrete.

The torsional prestressing wires were disposed helically round the axis of the ribs (*figs. 8* and *9*). Like the wires for producing the compressive and bending prestresses, they were fixed in position before the shuttering for the ribs was made. All wires (9 mm thick) were coated with a special preparation to prevent bonding with the concrete cast around them and to reduce friction losses when the wires were tensioned.

The contraction which a rib undergoes as a result of compressive prestressing is greater than the deformation, measured along the rib, suffered by the abutting shell under its own prestressing. If shell and rib were first to be joined firmly together and then prestressed, the difference in deformation would once again give rise to extra stresses in the shell. To avoid this, most of the ribs were partly prestressed before the shell was mounted, in such a way that when the final prestressing was applied the deformations then occurring in rib and shell were to all practical purposes equal.

Methods of prestressing

All wires were tensioned by a method devised and currently used successfully by Strabed. The main feature of this method is the possibility of tensioning wires between anchorings fitted to the structure in advance. With the traditional methods it would have been necessary to pull the wires through openings across the ribs, and this — apart from aesthetic objections — would have presented very considerable difficulties in the case of those beams where several wall surfaces meet at small relative angles.

In the method used, anchoring wires are cast into the ribs and into the substructure of the shells at ground level. Each of these wires is threaded through a small steel anchor plate and is crimped at each end. The prestressing wires are coupled to the anchoring wires by special sleeves (*fig. 10*), so designed that no slip can occur during this fixing process.

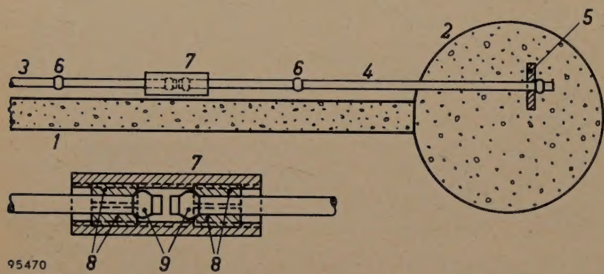


Fig. 10. The "Strabed" system for mounting prestressing wires. *1* = shell, *2* = rib (not shown in exactly the right proportions), *3* = prestressing wire. The anchoring wire *4* with anchorage plate *5* is fixed in the rib during fabrication of the latter. The wires are tensioned with a special jack which grips the crimps (*6*) in the prestressing wire and anchoring wire. The tensioned wire is clamped tight in an internally threaded coupling sleeve *7*, by screwing two split annulus pieces *8* against the crimped ends *9*.

This is particularly important for short wires which were necessarily relatively numerous in the pavilion. The wires are tensioned with a special jack designed by Strabed; this grips the wires from the side. The amount of tensioning applied can be read accurately from a dynamometer fitted to the jack (*fig. 11*).

As may be seen in fig. 11, it is possible, with this Strabed method, to lay the wires very close to the walls to be prestressed (in the Philips pavilion, only 2 to 3 cm from the shells). This is useful in order to be able to introduce the prestresses centrally in the shells, and moreover it is convenient for constructional reasons, since each prestressing wire must be fixed at regular points to the wall.

All the exterior surfaces of the pavilion, including the prestressing wires, were given a coating of special waterproof paint. To this base a final coating of aluminium paint was applied. *Fig. 12* shows the finished pavilion from the entrance end.

In view of the fact that pictures were to be projected on the interior walls, the prestressing wires on the inside shell surfaces were concealed by a sound-absorbent layer of asbestos felting.

Concluding remarks

The erection of exhibition buildings usually involves having to work against time, especially when the building departs so radically from classical concepts as the present one. The marks of haste are bound to be present in the finished structure; this need not be objectionable, however, since a temporary structure can be measured against different standards. Because of this circumstance we were able to experiment with an entirely new system of construction for the Philips pavilion, a system which we would otherwise probably not have dared to embark upon but which in this case was remarkably consistent with the ideas of the patron — N.V. Philips — and of the responsible architect — Le Corbusier. The Belgian Inspectorate of Works (Séco: Bureau de contrôle pour la Sécurité de la Construction en Belgique), by acting expeditiously

Fig. 11. Tensioning a prestressing wire with the "Strabed" jack. The tensioning force is read on a dynamometer.

Photo Hans de Boer

Fig. 12. The finished pavilion, seen from the entrance end.

and offering constructive criticism, also did a great deal to help make the experiment succeed. We were therefore able to use this unique opportunity to gather information and experience which, under normal circumstances, would only have been obtained in the face of much opposition and after a much longer period.

In a number of countries wide and varied use has already been made of shells composed of saddle surfaces, such as the hyperbolic-paraboloidal shell. They have been introduced, for example, as roofs for large halls. We are convinced that their construction in prestressed concrete, whether or not combined with prefabrication, offers yet more

interesting and wider possibilities. The Philips pavilion constitutes proof that such shells can be pieced together to form very intricate structures that serve both as roof and wall. Architects may find it rewarding to exploit the possibilities thus revealed.

Summary I-IV

Based on an idea of L. C. Kalff, the Philips pavilion was designed and equipped for automatic performances of a spectacle in light and sound, a so-titled "Electronic Poem", the scenario of which was written by Le Corbusier and the music composed by Edgar Varèse. The architectural design was undertaken by Y. Xenakis under the direction of Le Corbusier. Visual, acoustical and architectural considerations led to the pavilion being built up entirely from ruled surfaces, viz. hyperbolic paraboloids. In article I, an authorized shortened version of the original French text, the evolution of the first design, which also contained conoids, is illustrated by a series of original sketches by the architect. A description is then given of the semi-experimental, semi-geometrical method by which the surfaces of the first design were completely converted into hyperbolic paraboloids (second design). As a result of this conversion it was found possible to adopt the scheme of construction proposed by the contracting firm "Strabed", which was to build the pavilion as a self-supporting shell in prestressed concrete 5 cm thick.

The excellent mechanical properties of shells shaped as hyperbolic paraboloids (hypars) which made this scheme of construction possible, are discussed in general terms in the second article. After recapitulating the most important geometrical properties of the hyperbolic paraboloid, the author gives a concise explanation of the membrane theory of the hypar shell. The differential equations for the state of stress have an extremely simple solution in the case of a uniaxial, uniform load applied parallel to the axis of the shell; the shell is then found to be a structure of equal strength. The author also gives formulae for somewhat more complex loads, which are of importance under practical conditions, and describes a simple graphical method of determining the principal shell forces and the prestressing forces required in order to avoid in the shell any tensile stresses (which concrete cannot withstand). Finally edge disturbances are considered that occur when the boundary conditions are such as to prevent the existence of a purely membrane state of stress. Some details are given of the stability of hypar shells against buckling and second-order buckling.

The theory gives a general idea of the behaviour of hypar shells and allows solutions to be found to certain detail problems, but a complete calculation of the mechanical properties of such a complex structure as the Philips pavilion is out of the question. For this purpose model tests were required, and these are described in article III. In accordance with the architects' design, a 1 : 25 scale model was made of the pavilion in plaster of Paris on a framework of wire-gauze, steel tubes simulating the ribs of the structure. The states of stress produced in the model under different loads (dead-weight, wind, etc.) were investigated with a large number of strain gauges and displacement gauges. The conclusion was that a structure of 5 cm thick concrete with 40 cm thick ribs would be sufficiently strong. In a second model 1 : 10, two of the hypar shells were built up from several hundreds of appropriately shaped plywood slabs. These shells were prestressed, and the results observed proved the feasibility of the proposed system of construction, using precast concrete slabs and prestressing wires anchored in the ribs.

This scheme of construction, proposed and translated into reality by the firm "Strabed", is described at some length in the fourth article. The consideration underlying this scheme of construction was that the irregular and steeply twisted shells of the pavilion, which Strabed decided to keep as thin as possible (subject to the minimum of 5 cm dictated by acoustic insulation demands) could not be made by casting concrete *in situ* in shuttering. The hypar shells were therefore built up from more than 2000 slabs, each about 1 m^2, which were cast in open sand-bed moulds, built to the required shape. The slabs were subsequently erected on site on scaffolding, and pulled tight by steel wires. This construction is also mechanically favourable in that with suitable prestressing of shells and edge members (ribs) the structure can be brought approximately into a membrane state of stress. The precasting and assembly of the slabs were greatly simplified by the fact that the walls were ruled surfaces. The prestressing wires (applied on both surfaces of the walls) followed, as a rule, the ruling lines of the hyperbolic paraboloids. The wires were tensioned using a system devised by Strabed. The special feature of this system is that the anchorings for the wires are fitted to the structure in advance; this apparently minor detail had an important bearing on the successful erection of the pavilion. In addition to compressive and bending prestressing, some ribs were given torsional prestressing, partly to compensate for the torsional moments exerted on the ribs by abutting shells, and partly to introduce the prestressing forces centrally in the shells. This is thought to be the first application of torsional prestressing to concrete.
